



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: 20/1/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS)

- a) Define a second order linear partial differential equation (4 marks)
- b) State the short form a second order partial differential equation (3 marks)
- c) State the three classes of the second order partial differential equation (4 marks)
- d) Consider the equation $2u_{xx} + 3u_{xy} + u_{yy} = 0$.
 - i. Classify the equation (4 marks)
 - ii. Determine its characteristic equation (5 marks)
 - iii. Transform the equation to canonical form (7 marks)
 - iv. Solve the equation (7 marks)

QUESTION TWO (20 MARKS)

Prove the general equation $A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu = 0$

Where A, B, C, D, E, F are real constants such that $A \neq 0$.

QUESTION THREE (20 MARKS)

Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

Given that $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$.

QUESTION FOUR (20 MARKS)

Determine the characteristic of the Tricomi equation $y u_{xx} + u_{yy} = 0$ in the lower-half plane $y < 0$.

Put this equation into canonical form in the upper half- plane $y > 0$.

QUESTION FIVE (20 MARKS)

- a) An infinitely long string having one end at $x = 0$ is initially at rest on the x -axis. At $t = 0$;the end $x = 0$ begins to move along the u -axis in a manner described by $u(0, t) = a \cos \sigma t$. Determine the displacement $u(x, t)$ of the string at any point at any time. (10 marks)
- b) Using Mong's method obtain the integral of $q^2 r - 2 p q s + p^2 t = 0$ in the form $y + x f(z) = F(z)$ (10 marks)