MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 462: OPERATION RESEARCH III

DATE: 30/4/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the areas of application of integer programming (3 marks)

b) Explain the necessity of goal programming (3 marks)

c) Determine whether the function is convex, concave or neither:-

$F(X) = {x_1}^2 + {x_2}^2 - 2x_1x_2$ (3 marks)

d) Construct the dual of the problem

$$Minimize\ \ Z = x_2 + 3x_3$$

$$\begin{aligned} & 2x_1 + x_2 \le 3 \\ Subject\ to\ & x_1 + 2x_2 + 6x_3 \ge 5 \\ & -x_1 + x_2 + 2x_3 = 2 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

(4 marks)

e) Solve the non-linear programming problem using Lagragean multiplier

$$Maximize\quad Z = 5x_1 + x_2 - (x_1 - x_2)^2$$

$$Subject\quad \begin{aligned} & x_1 + x_2 = 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(6 marks)

f) Solve the NLPP using Kunh-Tucker conditions

$$Maximize\quad Z = 10x_1 + 4x_2 - 2{x_2}^2 - {x_2}^2$$

$$Subject\quad \begin{aligned} & 2x_1 + x_2 \le 5 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(6 marks)

g) A company manufactures two products, radios and transistors, which must be processed through assembly and finishing departments. Assembly has 90 hours available, finishing can handle up to 72 hours of work. Manufacturing one radio requires 6 hours in assembly and 3 hours in finishing. Each transistor requires 3 hours in assembly and 6 hours in finishing. If profit is Rs.120 per radio and Rs.90 per transistor. Formulate a goal programming for this situation so as to realize a profit of 2,100 (6 marks)

QUESTION TWO (20 MARKS)

a) Use Lagrangian multiplier method to solve the NLPP:-

$$Minimize \quad Z = {x_1}^2 + {x_2}^2 + {x_3}^2$$

$$Subject\ to \quad 4x_1 + {x_2}^2 + 2x_3 = 14$$

$$x_1, x_2 \geq 0$$

(8 marks)

b) Solve using duality

$$Minimize \quad Z = 0.7x_1 + 0.5x_2$$

$$x_1 \leq 4$$
$$x_2 \geq 6$$
$$x_1 + x_2 \geq 0$$
$$2x_1 + x_2 \geq 18$$
$$x_1, x_2 \geq 0$$

(12 marks)

QUESTION THREE (20 MARKS)

Use Gomory's cutting plane algorithm to solve

$$Maximize \ Z = x_1 - x_2$$

$$Subject\ to \quad \begin{array}{l} x_1 + 2x_2 \leq 4 \\ 6x_1 + 2x_2 \leq 9 \\ x_1, x_2 \geq 0 \ and\ are\ integers \end{array}$$

(20 marks)

QUESTION FOUR (20 MARKS)

A company manufactures two products, radios and transistors, which must be processed through assembly and finishing departments. Assembly has 90 hours available, finishing can handle up to 72 hours of work. Manufacturing one radio requires 6 hours in assembly and 3 hours in finishing. Each transistor requires 3 hours in assembly and 6 hours in finishing. If profit is Rs.120 per radio and Rs.90 per transistor. Determine the best combinations of radios and transistors to realize a profit of Rs.1500 and to meet the demand of 10 radios, obtain the optimal solution

QUESTION FIVE (20 MARKS)

a) Obtain the necessary and sufficient conditions for the optimal solution of the following problem. What is the optimal solution?

$Minimize \quad Z = 2e^{3x_1+1} + e^{2x_2+3}$

$Subject\ to \begin{array}{c} x_1 + x_2 = 5 \\ x_1, x_2 \geq 0 \end{array}$ (8 marks)

b) Consider the following function:-

$f(X) = 5x_1 + 2x_2 + x_3{}^2 - 3x_3x_4 + 4x_4{}^2 + 2x_5{}^4 + x_5{}^2 + 3x_5x_6 + 6x_6{}^2 + 3x_6x_7 + x_7{}^2$ show that $f(X)$ is convex by expressing it as a sum of functions of two variables and then proving that all the functions are convex (12 marks)