MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

EEE 402: COMPLEX ANALYSIS FOR ENGINEERING

DATE: 29/3/2021 **TIME: 11.00-1.00 PM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

Programmable calculators are prohibited

QUESTION ONE (COMPULSORY) (30 MARKS)

a). Define the following terms

i. A complex number (1 mark)

ii. Analytic function (1 mark)

b). Test the continuity of a complex function

$$f(z)=\begin{cases}\dfrac{z^2+4}{z-2i} & ,z\neq 2i\\ 3+4i & ,z=2i\end{cases}$$ (4 marks)

c). Determine the singular points of the following function and residues at each point.

$$f(z)=\frac{z^2}{(z-1)^2(z+2)}$$

and hence evaluate $\int_c \frac{z^2}{(z-1)^2(z+2)}dz$ where $C:|z|=3$. (6 marks)

d). Find all values of $(i)^{1+i}$ hence state the principle value (3 marks) e). Evaluate $\int_C \frac{z+4}{z^2+2z+5}dz \; C:|z+1|=3$, using Cauchy's theorem (5 marks)

f). Let $u(x,y)=x^2-y^2-2xy-2x-y-1$. Find $v(x,y)$ such that $f(z)=u(x,y)+iv(x,y)$ is harmonic. Express $f(z)$ in terms of z. (6 marks)

g). Evaluate $\int_0^{1+i}(x-y+ix^2)dz$ along the straight line from $z=0$ to $z=1+i$ (4 marks)

QUESTION TWO (20 MARKS)

a) Find all values of z such that $e^z=1+i\sqrt{3}$ (6 marks)

b) Show that $\cos^{-1}(z)=-i\ln\left(z+i\sqrt{1-z^2}\right)$. Hence find all solutions to the equation $\cos z=\sqrt{2}$

(7 marks)

b) Show that $|\cosh z|^2=\cos^2 y+\sinh^2 x$ (7 marks)

QUESTION THREE (20 MARKS)

a) Derive the Cauchy-Riemann conditions for analytic function. (7 marks)

b) Show that $f(z)=\ln z$ is analytic function (6 marks)

c) Show that $v(x,y)=\ln(x^2+y^2)+x-2y$ is harmonic. Find $u(x,y)$ such that $u(x,y)+iv(x,y)$ is analytic. Express $f(z)$ in terms of z. (7 marks)

QUESTION FOUR (20 MARKS)

a) Let $f(z)$ be analytic in the simply connected region R, let C be a simple closed contour that lies in R. If $z=a$ is a point that lies interior to C, show that

$$f''(a)=\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^3}dz.$$ (7 marks)

b) Evaluate the following integral $\int_c \frac{z-1}{(z+1)^2(z-2)}dz$ using Cauchy's integral formula where $c:|z-i|=2$. (7 marks)

c) Use De-Moivres theorem to show that $\cos 4\theta=8\sin^4\theta-8\sin^2\theta+1$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Find the fifth roots of the complex number $-4+4i$. (7 marks)

b) Evaluate $\int_{1-i}^{2+i}(2x+iy+1)\,dz$ along the curve $x=t+1$, $y=2t^2-1$ (6 marks)

c) Evaluate the following integral $\int_c \frac{12z-7}{(z-1)^2(2z+3)}\,dz$ using Cauchy's residue theorem where $c:|z+i|=\sqrt{3}$. (7 marks)