MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SMA 466: MULTIVARIATE ANALYSIS

DATE: 25/9/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Attempt Question One and any other TWO questions

QUESTION ONE (30 Marks)

a) Suppose $\underline{X} = (x_1 \quad x_2 \quad x_3)^|$

$$\mu_3 = (\mu_1, \mu_2, \mu_3)^| \text{ and } \Sigma = \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{pmatrix}$$

If $\underline{X}$ is $N_3[\underline{\mu}, \Sigma]$Show that $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ given x_3 has a multivariate normal distribution with mean $\underline{r}$ and covariance matrix V where

$$\underline{r} = (\mu_1 + \rho^2(x_3 - \mu_3), \mu_2)^| \quad \text{and} \quad V = \begin{pmatrix} 1-\rho^4 & \rho \\ \rho & 1 \end{pmatrix}$$ (7 marks)

b) i) Describe the Wishart distribution,

$W_p = [\Sigma, n]$

ii) Let the random vector variable $\underline{Z}_{p\times1}$ have the multivariate Normal distribution with mean $\underline{O}_{p\times1}$, and covariance matrix I_{pxp} where I_{pxp} is a p by p identity matrix.

Determine the distribution of $Z^T Z$ and hence or otherwise show that

$$W_p[I_{pxp}, P] = \chi^2_{(p)}$$

. (8 marks)

c) Let $\underline{X}$ be a px1 random vector and $\underline{x}_1$, $\underline{x}_2$,………., $\underline{x}_n$ be a random sample of size n from the population.

Describe (i). $\underline{X}_r$ for some r = 1, 2,……..,n

(ii). The sample mean vector $\underline{\bar{X}}$

(iii). The sample dispersion matrix S.

(iv). The data matrix X

(8 marks)

d) Using the notations in part (a), show that

i) $E(\underline{\bar{X}}) = \underline{\mu}$ and

ii) $E(S) = \frac{n-1}{n} \Sigma$ where $\underline{\mu}$ and Σ denote the population mean vector and variance-covariance vector respectively. (7 marks)

QUESTION TWO (20 MARKS)

a) Let $\underline{X}$ be a p-component random vector which has the multivariate distribution, $N_p[\underline{\mu}, \sum]$. Show that for the sample mean vector $\underline{\bar{X}}$ and sample variance S;

i) $\underline{\bar{X}} \sim N_p\left[\underline{\mu}, \frac{\sum}{n}\right]$ (4 marks)

ii) $S \sim W_p\left[\frac{\sum}{n}, n-1\right]$ (4 marks)

iii) $\frac{n-p}{p}\left(\underline{\bar{X}} - \underline{\mu}\right)^1 S^{-1}\left(\underline{\bar{X}} - \underline{\mu}\right) \sim F(p,n\text{-}1)$ (4 marks)

b) Let $\underline{Y}$ be a random vector with mean $\underline{\mu}$ and covariance matrix $\Sigma = I_{pXp}$. Show that if $\underline{Y}$ is $N_p[\underline{\mu}, \Sigma]$, then $E\{(\underline{Y} - \underline{\mu})^1(\underline{Y} - \underline{\mu})\} = p$ (8 marks)

QUESTION THREE (20 MARKS)

a) Show that if $\underline{Y}$ is $N_p[\underline{\mu}, \Sigma]$ then an rx1 subvector of $\underline{Y}$ has an r-variate normal distribution with the same means, variances and covariances as in the original p-variate normal distribution.

(6 marks)

b) Using part (a) , or otherwise, Show that any individual variable Y_i in $\underline{Y}$ is distributed as $N_p[\mu_i, \sigma_{ii}\ ,]$ where $\mu_i = E(Y_i)$, $\sigma_{ii} = V(Y_i)$ (6 marks)

c) Show that if $\underline{Y}$ and $\underline{X}$ are jointly multivariate normal with $\Sigma_{yx} \neq 0$, then the conditional distribution of $\underline{Y}$ given as $\underline{X}$, $f(\underline{Y}/\underline{X})$ is multivariate normal with mean vector and covariance matrix.

$$E(\underline{Y}/\underline{X}) = \underline{\mu}_y + \Sigma_{yx} \Sigma_{xx}^{-1}(\underline{X} - \underline{\mu}_x)$$

$$V(\underline{Y}/\underline{X}) = \Sigma_{yy} - \Sigma_{yx} \Sigma_{xx}^{-1}\Sigma_{yx}$$

respectively, given corresponding covariance matrices (8 marks)

QUESTION FOUR (20 MARKS)

Let the random vector $\underline{V}$ be $N_4[\underline{\mu}, \Sigma]$ where $\underline{\mu} = (2\ \ 5\ -2\ \ 1)^1$, and

$$\Sigma = \begin{pmatrix} 9 & 0 & 3 & 3 \\ 0 & 1 & -1 & 2 \\ 3 & -1 & 6 & -3 \\ 3 & 2 & -1 & 7 \end{pmatrix}$$

Let $\underline{V} = (Y_1\ Y_2\ \ X_1\ X_2)$ (6 marks)

a) Determine the distribution for $(Y_1\ Y_2)^1 / (X_1\ X_2)^1$

b) Hence or otherwise, calculate

i) $Cov(X_1, X_2)$

ii) $Cov(\ (X_1\ X_2)^1 / (Y_1\ Y_2)^1\ )$

iii) Comment on the results in b(i) and b(ii) (8 marks)

c) Find the partial correlation matrix

(corr ($(X_i\ X_j)^1 / (Y_1\ Y_2)^1$) for i,j=1,2 (6 marks)

QUESTION FIVE (20 MARKS)

Consider a random vector $x^1 = (x_1 \quad x_2)$. Assume x has $N_2\,[\mu, \Sigma]$ distribution. A sample of size n = 25 has $\underline{\overline{X}} = (185.72,\ 183.84)^1$

a) Suppose Σ is unknown and the sample dispersion matrix

$$S = \begin{pmatrix} 91.481 & 66.875 \\ 66.875 & 96.775 \end{pmatrix}$$

Test $\quad H_0 : \Sigma = \Sigma_0 \quad$ where $\Sigma_0 = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$

Assume 5% level of significance. (10 marks)

b) Suppose now $\Sigma = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$ while the mean vector $\underline{\mu}$ is unknown.

Test

$H_0 : \mu = \mu_0$

where $\mu_0 = (182 \quad 182)^1$ at 5% level of significance. (10 marks)

c). Suppose that B is a kxp matrix of constants. A is a pxp symmetric matrix of constants, and $\underline{Y}$ is distributed as $N_p\,[\underline{\mu}, \Sigma]$.

Show that $B\underline{Y}$ and $\underline{Y}^1 A \underline{Y}$ are independent if and only if $B \Sigma A = 0$

(10 marks)

d). Let A and B be symmetric matrices of constants. If $\underline{Y}$ has $N_p\,[\underline{\mu}, \Sigma]$ distribution, show that $\underline{Y}^1 A \underline{Y}$ and $\underline{Y}^1 B \underline{Y}$ are independent if and only if

$A \Sigma B = 0$ (10 marks)

QUESTION FOUR (20 MARKS)

a). Let $\underline{X}$ be the p-component vector describe in Question ONE. If

(i). $\underline{X} \sim N_p\left[\underline{\mu}, \frac{\sum}{n}\right]$ (4 marks)

(ii). $S \sim W_p\left[\frac{\sum}{n}, n-1\right]$ (4 marks)

(iii). $\frac{n-p}{p}\left(\overline{X}-\underline{\mu}\right)^1 S^{-1}\left(\overline{X}-\underline{\mu}\right)$~F(p,n-1) (4 marks)

b). Let $\underline{Y}$ be a r.u. with mean $\underline{\mu}$ and covariance matrix $\sum$. Show that if $\underline{Y}$ is N_p = [μ, Σ], then Var $(A^1\underline{Y}A) = 2 \text{ trace } \{(A\ \Sigma\ )^2\} + 4\underline{\mu}^1 A\underline{\mu}$

(8 marks)