MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 408: LINEAR ALGEBRA IV

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given that $\boldsymbol{u} = (i, 3-i)$ $\boldsymbol{v} = (2+i, 3+i)$ and $\boldsymbol{w} = (4i, 6)$ perform each of the indicated operations

i. $\boldsymbol{u} + i\boldsymbol{v} + 2i\boldsymbol{w}$ (2 marks)

ii. $(6+3i)\,\boldsymbol{v} - (2-2i)\boldsymbol{w}$ (2 marks)

b) Determine the Euclidean norm of $\boldsymbol{v} = (1,\ 2+i, -i)$. (3 marks)

c) Find the Euclidean distance between $\boldsymbol{u}$ and $\boldsymbol{v}$ where

$\boldsymbol{u} = (1, 2, 1, -2i)$ and $\boldsymbol{v} = (i,\ 2i,\ i, 2)$ (3 marks)

d) Determine whether the following set of vectors form a basis of $\mathbb{C}^3$:

$\{(1+i, 1-i, 1),\ (i, 0, 1),\ (-2, -1+i, 0)\}$ (3 marks)

e) Show that the matrix

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & i & 0 \\ \frac{1+i}{2} & 0 & \frac{-1-i}{2} \end{bmatrix}$$

is unitary. (3 marks)

f) Find the determinant of the Hermitian matrix $G = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix}$. (3 marks)

g) Find the eigenvalues and the corresponding eigen vectors of the matrix $T = \begin{bmatrix} 2 & 0 & -i \\ 0 & 3 & 0 \\ i & 0 & 2 \end{bmatrix}$ (4 marks)

h) Find the symmetric matrix which corresponds to the quadratic polynomial

$Q(x, y, z) = 3x^2 + 4xy - y^2 + 8xz - 6yz + z^2$. (2 marks)

i) i) Define what is meant by a bilinear form. (2 marks)

ii) Define $H: R^2 \times R^2 \to R$ by $H(x, y) = 2x_1y_1, +3x_1y_2 + 4x_2y_1 - x_2y_2$

where$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ are in R^2.

Verify that H is a bilinear form on R^2. (3 marks)

QUESTION TWO (20 MARKS)

a) Perform the given operation using

$A = \begin{bmatrix} 4-i & 2 \\ 3 & 3+i \end{bmatrix}, B = \begin{bmatrix} 1+i & i \\ 2i & 2+i \end{bmatrix}$

i) $3BA$ (3 marks)

ii) $\det(A - B)$ (3 marks)

b) Find a unitary matrix P such that P^*AP is a diagonal matrix

Where $P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1+i \\ 0 & -1-i & 0 \end{bmatrix}$ (6 marks)

c) Prove that every Hermitian matrix P can be written as the sum $P = A + iB$ where A is a real symmetric matrix and B is a real skew- symmetric. (5 marks)

d) Hence write the matrix P in (b) above in the form $P = A + iB$ shown in (c). (3 marks)

QUESTION THREE (20 MARKS)

a) Give the definition of a complex inner product space or unitary space (4 marks)

b) Determine whether the function defined by $< \boldsymbol{u}, v > = 4u_1v_1 + 6u_2v_2$ where $\boldsymbol{u} = (u_1, u_2)$ and $\boldsymbol{v} = (v_1, v_2)$ is a complex inner product. (6 marks)

c) Let V be a finite dimensional inner product space and let S and T be linear operators on V show that:

i) $< x, (T + S)^* y > = < (T + S)x, y)$ (4 marks)

ii) If T is a normal operator show that $\|Tx\| = \|T^*x\| \ \forall \ x \in V$ (2 marks)

d) Define $T: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ by

$Tx = T(x_1x_2) = (2ix_1 + 3x_2, \ x_2 - x_2)$ if $B = \{e_1e_2\}$ is the standard basis for $\mathbb{C}^2$, find $[T^*]_B$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Let $H = \begin{bmatrix} 1 & 1+i & 2i \\ 1-i & 4 & 2-3i \\ -2i & 2+3i & 7 \end{bmatrix}$ a Hermitian matrix

find a non-singular matrix P such that $P^T HP$ is diagonal. (5 marks)

b) Let $V = R^n$ be a vector space. Give the definition of a quadratic form on $= R^n$. (2 marks)

c) Define $Q(x) = 5x_1^2 + 3x_2^2 + 2x_3^2 - x_1x_2 + 8x_2x_3$for $x \in R^3$.

i. Express this quadratic expression in the form $X^T AX$ (4 marks)

ii. Find the matrix A. (5 marks)

d) By writing $x = P_y$or $y = P^{-1}x$, transform the quadratic form in (c)above such that it does not have cross-products x_1x_2 and x_2x_3. (4 marks)

QUESTION FIVE (20 MARKS)

a) State the Cayley Hamilton theorem, hence use the matrix $A = \begin{bmatrix} 3 & 1-i \\ 1+i & 2 \end{bmatrix}$ to support your definition. (4 marks)

b) Differentiate between Jordan canonical form and Rational form of an operators T.(2 marks)

c) Find all possible Jordan Canonical form of a matrix whose minimum polynomial is $m(t) = (t-2)^2(t-3)^2$. (4 marks)

d) Let f be the bilianear for on R^2 defined by $f(< x_1\, x_2 >< y_1\, y_2 > = 2x_1y_1 - 3x_2y_2 - x_2y_2$. Find:

i. the matrix A of f in the basis $\{\boldsymbol{u_1} = (1,0),\ \boldsymbol{u_2} = (1,1)\}$. (3 marks)

ii. the matrix B of f in the basis $\{\boldsymbol{v_1} = (2,1),\ \boldsymbol{v_2} = (1,-1)\}$. (3marks)

iii. the change of basis matrix P from the basis $\{\boldsymbol{u_i}\}$ to the basis $\{\boldsymbol{v_i}\}$ and show that $B = P^T AP$. (4 marks)