



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF BUSINESS, ECONOMICS AND HOSPITALITY AND TOURISM

MANAGEMENT

DEPARTMENT OF ECONOMICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS

EES 300: MATHEMATICS FOR ECONOMISTS III

DATE:

TIME:

INSTRUCTIONS

Answer Question **ONE** and any other **TWO** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) In a laboratory experiment, the rate of change of population of bacteria $P(t)$ with time (t) is given by the following $\frac{dP(t)}{dt} = 0.045[900 - P(t)]$

Given that $P(0) = 0$, find the following;

- The time path $P(t)$ (1 mark)
 - The rate of growth of the population of bacteria at $t = 0$ and at $t = 10$ (1 mark)
 - The steady state (inter-temporal equilibrium value of $P(t)$) (2 mark)
 - Provide the graphical sketch of time path of the population of bacteria (3 marks)
- b) Given the following constrained optimization problem
- Maximize $Z = 2x + 6y - 3y^2$
- Subject to $2x - 2y^2 = 8$
- Find the critical values of x and y (2 marks)
 - Do the critical values present a maximum or a minimum? Apply Bordered Hessian method. (2 marks)
- c) Explain the following terms

- i. Dynamic stability
 - ii. Steady state (2 marks)
- d) Discuss the dynamic stability of the following time-path
- i. $y(t) = 5e^{-3t} + 15$
 - ii. $y(t) = 3e^{-at} + 5$ where $a = -2$
 - iii. $y(t) = 40^{-0.3t} + 8$ (4.5 marks)
- e) A monopolistic firm has the following demand and total cost functions
- $$P = 100 - 2Q$$
- $$TC = \frac{1}{2}Q^3 - 9Q^2 + 120Q + 175$$

Required

- i. the TR and π functions (2 marks)
 - ii. the profit-maximizing level of Q. Confirm the second order condition is satisfied (1.5 mark)
 - iii. the MC and MR at the profit-maximizing level of Q. Compare their magnitudes (2 marks)
 - iv. the value of Q that maximizes TR. Confirm that the second order condition is satisfied (2 marks)
- f) Solve for the following:
- $$dy/dt + 10y = 20 \text{ given } y(0) = 10 \quad (5 \text{ marks})$$

QUESTION TWO (20 MARKS)

- a) Consider the following demand function

$$Q_a = 300 - 3P_a + 2P_r + 0.2Y \text{ Where } P_a = 20; P_r = 30 \text{ and } Y = 800$$

Determine the following and interpret the results

- i. Own price elasticity of demand (2 marks)
 - ii. Cross price elasticity of demand (2 marks)
 - iii. Income elasticity of demand (2 marks)
 - iv. Are the two goods (a and r) substitutes or compliments explain (2 marks)
 - v. Is good a normal or inferior good? Explain (2 marks)
- b) Find the area
- i. between $f(x)$ and $g(x)$, such that $f(x) = \sqrt{x}$ and $g(x) = x^3$ (3 marks)
 - ii. under the curve $f(x) = 2xe^{x^2}$ from $x = 0$ to $x = 5$. (3 marks)
- b) Find the following indefinite integral: $\int \frac{1}{x^2+x-6} dx$ (4 marks)

QUESTION THREE (20 MARKS)

- a) Given the following bivariate equations, compute the partial derivatives, second-order partial derivatives and cross partial derivatives. Comment on the cross partial derivatives

i. $Z = f[x, y] = [3x^3 + 4y^6 - y^4]^8$ (3 marks)

ii. $Z = \left[\frac{4x^2+7}{x^2+y^2} \right]$ (3 marks)

- b) Explain the term returns to scale (4 marks)

- c) Find the determinant of

$$[A] = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \quad (5 \text{ marks})$$

- d) Find the inverse of A and the subsequent determinant

$$[A] = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -2 \\ 3 & 4 & -1 \end{bmatrix} \quad (5 \text{ marks})$$

QUESTION FOUR (20 MARKS)

- a) The demand and supply equations for the cob web model are given respectively as follows: $Q_{dt} = 3 - 0.3P_t$ and $Q_{st} = -2 + P_{t-1}$. Find the time path for P_t , and check whether it is stable or not (10 marks)

- b) The equation of motion for capital (K) is given as:

$$\frac{\partial K}{\partial t} = I_o - \delta K$$

Where I_o represent the constant level of investment and δ represents rate of depreciation. Find the capital stock at time t $[K(t)]$ and explain whether $K(t)$ converges to a steady state (10 marks)

QUESTION FIVE (20 MARKS)

- a) At a price of Sh. 5 per unit, a firm will supply 5000 units of a product every week. But when the price drops to Sh. 3.50 the supply also drops to 2000 units. Determine the equation of the supply function. (10 marks)
- b) A firm has analyzed its sales and found that its customers will buy 20% more units of its products with each Sh. 2.00 reduction in price. When the price is Sh. 12.00, the firm sells 500 units. What is the equation of the demand function for this product? (10 marks)