# MACHAKOS UNIVERSITY

**University Examinations for 2021/2022 Academic Year**

**SCHOOL OF ENGINEERING AND TECHNOLOGY**

**DEPARTMENT OF MECHANICAL AND MANUFACTURING ENGINEERING**

**FOURTH YEAR SECOND SEMESTER EXAMINATION FOR**

**BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)**

**EMM 403: SOLID AND STRUCTURAL MECHANICS 111**

**DATE:** **TIME:**

---

**INSTRUCTIONS**

This paper contains FIVE questions

Question ONE is **compulsory** and carries 30 Marks.

Questions TWO – FIVE carries 20 Marks each. **Answer any Two.**

**QUESTION ONE (COMPULSORY) (30 MARKS)**

a) i) How will you know if a given cylinder is thick or thin? (1 mark)

ii) A cylinder has an inside diameter of 150 mm and outside diameter of 450 mm. If it is subjected to an internal pressure of 160 MPa and external pressure of 80 MPa, determine the maximum radial, tangential and shear stresses. The ends are closed. (4 marks)

b) i) What is critical load as applied in failure of columns and struts? (1 mark)

ii) Derive the Euler's equation that can be used to determine the crippling load of the column loaded as shown in Fig Q 1 b ii). (6 marks)

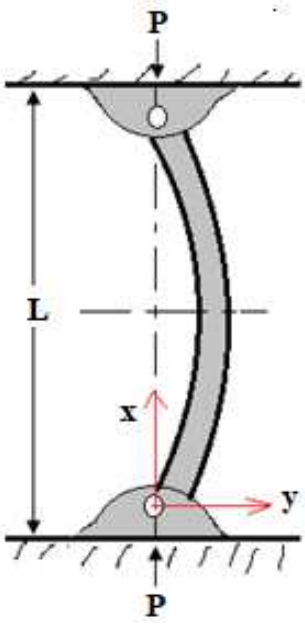


Fig. Q 1 b ii)

c) A load of 10MN is supported at B by two rods of the same material and same diameter 10 mm. Determine the horizontal and vertical deflections of point B using Castigiliano's theorem. Take E = 200 GPa (5 marks)

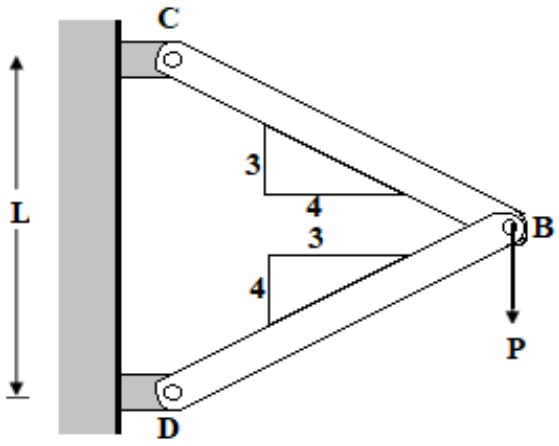


Fig. Q1 c)

d) i) Differentiate between plates and sheets (2 marks)

ii) Consider a simply supported rectangular plate of sides $a$ $x$ $b$ and thickness $t$. The plate carries a uniformly distributed load $P$ per unit area over the entire surface. Show that the plates maximum bending stress will be given by

$$\sigma_{max} = \frac{Pa^2b^2}{2t^2\sqrt{(a^2+b^2)}}$$ (6 marks)

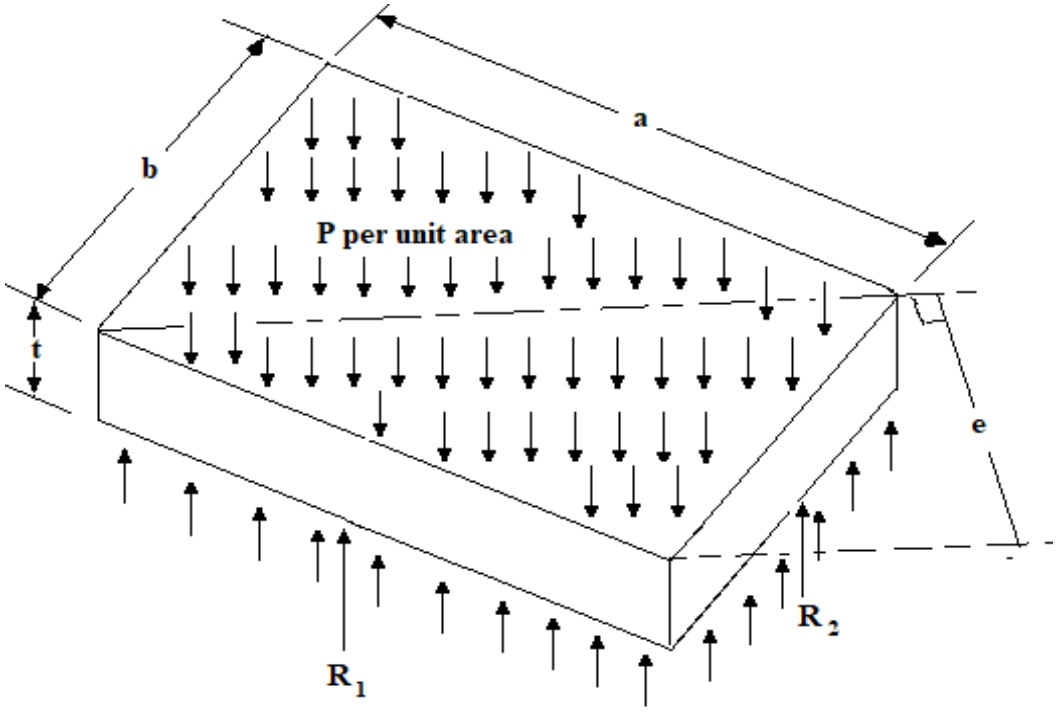


Fig. Q1 d ii)

e) i) State one example of a rotating thin disc (1 mark)

ii) A thin disc of mass 50 kg, radius 60 mm and thickness 25 mm rotates at a uniform angular speed of 70 rad/s. If the disc is fabricated from a material whose Poisson's ration is 0.25, determine the radial and circumferential stresses developed at mid radius of the disc when in rotation. (4 marks)

**QUESTION TWO (20 MARKS)**

a) Show that the tangential and radial stresses for a thick-walled cylinder are given by the following Lames equation

$$\sigma_r = \left(\frac{a^2p_i - b^2p_o}{b^2 - a^2}\right) - \left(\frac{a^2b^2(p_i - p_o)}{b^2 - a^2}\right)\frac{1}{r^2}$$

$$\sigma_t = \left(\frac{a^2p_i - b^2p_o}{b^2 - a^2}\right) + \left(\frac{a^2b^2(p_i - p_o)}{b^2 - a^2}\right)\frac{1}{r^2}$$

Where

$p_i$ – internal pressure

$p_o$ – Outer pressure

$a$ – internal diameter

$b$ – Outer diameter (12 marks)

b) A pressure vessel of 0.3 m internal diameter, 0.4 m external diameter and 0.9 m long, with closed ends is to be subjected to a hydraulic test of 15 N/mm$^2$. Calculate the change of internal and external diameters. E = 210 GPa and ν = 0.3 (8 marks)

**QUESTION THREE (20 MARKS)**

a) Fig. Q3) show an eccentrically loaded strut. Show that the allowable stress in the strut is given by the following secant formula

$$\sigma_{all} = \frac{\sigma_{max}}{1 + \frac{eC}{r^2}\sec\frac{kL_e}{2}}$$

Where

C - the maximum distance from the neutral axis

r - Slenderness ratio

$l_e$ - effective length (12 marks)

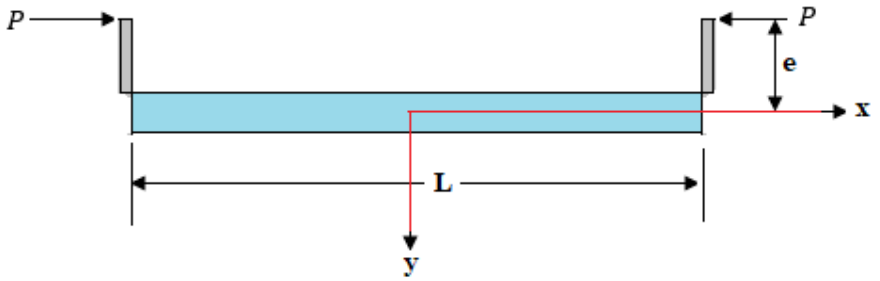


Fig. Q3 a)

b) An aluminium column is fixed at one end at the bottom and braced with cables at the top to prevent movement on the x-z plane. Determine the minimum value of P if a factor of safety of 2 is desired. (Take L = 6 m, E = 70 GPa, $\sigma_y = 215$ MPa, $A = 7.5 * 10^{-3}$ m$^2$, $I_x = 6.13 * 10^{-5}$ m$^4$, $I_y = 2.32 * 10^{-5}$ m$^4$) (8 marks)

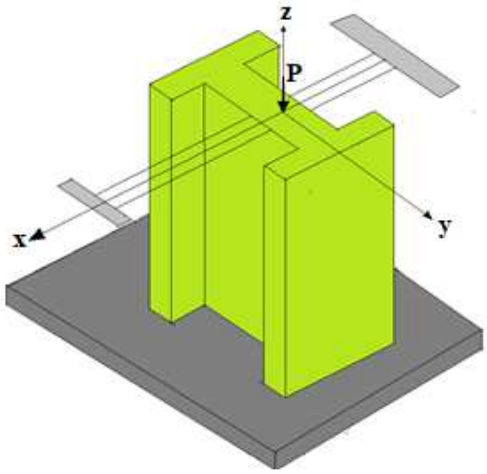


Fig Q 3 b)

**QUESTION FOUR (20 MARKS)**

Fig. Q 4) shows a circular plate of radius ***a*** subjected to a uniform pressure ***P*** or load ***P*** per unit area.

a) Derive an equation for deflection (***w***) of the plate and hence determine the maximum deflection in the plate. (12 marks)

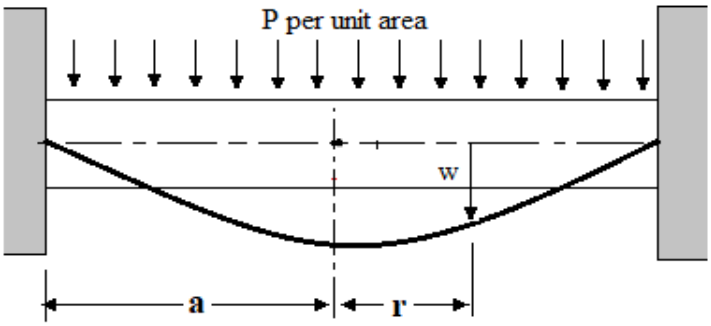


Fig. Q 4)

b) Determine the maximum bending stress in the plate in terms of ***P***, thickness ***t*** and plate radius ***a***. (8 marks)

**QUESTION FIVE (20 MARKS)**

a) The general equations for determination of circumferential and radial stresses of a thin disc rotating at an angular speed of $\omega$ are;

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2$$

$$\sigma_r = A - \frac{B}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2$$

where A and B are constants to be determined from boundary conditions for the particular problem.

Consider a circular disc of uniform thickness, internal radius $R_i$ and outer raius $R_o$, rotating at a uniform angular speed $\omega$, determine the expressions for maximum and mimium circumferential and radius stresses and hence sketch the stresses variations in the disc.

(12 marks)

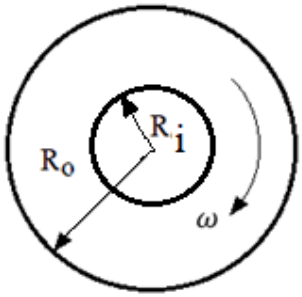


Fig. Q 5a)

b) A constant strength disc of diameter 950 mm and peripheral thickness 30 mm rotates at a constant speed of $300\pi$ rad/s. The maximum stress, $\sigma_{max}$, at any point of the disc is not to exceed 900 MN/m$^2$. Starting from the equilibrium equation;

$$\sigma_\theta - \frac{1}{t}\frac{d}{dr}(r.t.\sigma_r) - \rho\omega^2 r^2 = 0$$

determine the minimum thickness of the disc at the axis of rotation.

Assume density of the disc material is $\rho = 7.83 \times 10^3$ kg/m$^3$ (8 marks)