



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN
MATHEMATICS

SMA 304: NUMBER THEORY

DATE: 26/7/2017

TIME: 8:30 – 10:30 AM

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) If a / b and c / d show that $a c / b d$. (5 marks)
- b) Show that $\sqrt{11}$ is irrational. (5 marks)
- c) The square of an odd integer is odd and the square of an even integer is even proof. (6 marks)
- d) Is 117 a prime number? (4 marks)
- e) Show that $1 + 5 + 5^2 + \dots + 5^{11}$ is composite. (5 marks)
- f) Show that if p is prime and the $GCD(a, p) > 1$, then p/a . (5 marks)

SECTION B: ANSWER ANY OTHER TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Prove that every odd number of either of the form $4p + 1$ or $4p + 3$. $p \in \mathbb{Z}$. (5 marks)

- b) Prove that for integer $n, n + 2, n + 4$ then one of them is divisible by 3. (5 marks)
- c) Use Eratosthene's sieve to find the prime numbers less than 90. (5 marks)
- d) Show that we cannot have 3 consecutive odd numbers other than 3,5,7 such that they are all prime. (5 marks)

QUESTION THREE (20 MARKS)

- a) Show that whenever d is the GCD of a and b (a, b) then $-d$ is also the GCD of a and b . (5 marks)
- b) Show that there are only 2 GCDs for any two pairs a and b . (5 marks)
- c) Express the GCD of (128,30) in the form $n(128) + m(30)$. (5 marks)
- d) Prove that if $a, b, c, d \in \mathbb{Z}$ and n is a positive integer and $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$. (5 marks)

QUESTION FOUR (20 MARKS)

- a) State Fermat's little theorem. (2 marks)
- b) Use Fermat's little theorem to obtain $3^{60} \pmod{61}$. (5 marks)
- c) By considering $f(x, y) \pmod{4}$ show that $f(x, y) = y^2 - x^2 - 2 = 0$ has no solution. (6 marks)
- d) Express $\sqrt{7}$ as continued fraction. (7 marks)

QUESTION FIVE (20 MARKS)

- a) State the Chinese remainder Theorem. (2 marks)
- b) If a / b and c / d is it true that $a + c / b + d$. (5 marks)
- c) Find $\phi(60)$ (3 marks)
- d) Find the set of solution z to the congruence $3z + 4 = 6 \pmod{7}$. (5 marks)
- e) Find the GCD of the two numbers (37129,14659) using Euclidean algorithm. (5 marks)