



MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 203: ACTUARIAL MATHEMATICS I

DATE: 9/8/2021

TIME: 11:00 – 1:00 PM

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Define the following actuarial notations.
- ${}_nq_x$ (2 marks)
 - ${}_nm_x$ (2 marks)
- b) An investment of £1,000 in an account accumulated to £2,500 after 5 years.
- State the accumulation factor $A(0,5)$ (2 marks)
 - Based on part (i)
 - Find the simple annual interest rate which would give the accumulation factor. (3 marks)
 - Find the annual compound interest rate which would give the accumulation factor (3 marks)
- c) Calculate
- ${}_{10|5}q_{40}$ (3 marks)
 - $\bar{a}_{65}$ (3 marks)
- Basis:
- Mortality AM92
 - Interest 4% per annum

- d) A population is subject to the force of mortality $\mu_x = e^{0.0002x} - 1$. Calculate the probability that a life now aged 20 exact:
- survives to age 70 exact (3 marks)
 - dies between ages 60 exact and 70 exact (4 marks)
- e) Calculate the present value on 1 September 2002 of payments of £280 due on 1 September 2004 and £360 due on 1 March 2005. Interest is 15% pa effective. .
(5 marks)

QUESTION TWO (20 MARKS)

- a) Proof that (9 marks)

$$\bar{a}_{x|y} = \frac{\bar{A}_{xy} - \bar{A}_y}{\delta}$$

- b) A life insurance company issues identical deferred annuities to each of 100 women aged 63 exact. The benefit is £5,000 per annum payable continuously from a woman's 65th birthday, if still alive at that time, and for life thereafter.
- Write down an expression for the random variable for the present value of future benefits for one policy at outset. (3 marks)
 - Calculate the total variance of the present value at outset of these annuities.
Using,
Basis:
 - Mortality: PFA92C20
 - Interest: 4% per annum
 (8 marks)

QUESTION THREE (20 MARKS)

- a) i) Give the difference between Term and Pure assurance. (4 marks)
- iii) At a certain company, the probability of each employee leaving during any given year is 5% independent of the other employees. Those who remain with the company for 25 years are given £ 1,000,000 . What is the expected present value of this payment to a new starter, assuming an interest rate of 7% p.a and ignoring the possibility of death? (6 marks)

- b) An investigation was carried out into the effects of lifestyle factors on the mortality of people aged between 50 and 65 years. The investigation took the form of a prospective study following a sample of several hundred individuals from their 50th birthdays until their 65th birthdays and collecting data on the following covariates for each person:

X_1	Sex (a categorical variable with 0 = female, 1 = male)
X_2	Cigarette smoking (a categorical variable with 0 = non-smoker, 1 = smoker)
X_3	Alcohol consumption (a categorical variable with 0 = consumes fewer than 21 units of alcohol per week, 1 = consumes 21 or more units of alcohol per week)

Table 1

In addition, data were collected on the age at death for persons who died during the period of investigation.

In order to analyse the data, it was decided to use a Gompertz hazard, $\lambda_x = BC^x$, where x is the duration since the start of the observation.

- Explain why the Gompertz hazard might be appropriate for analysing the mortality of persons aged between 50 and 65 years. (2 marks)
- Using the Gompertz hazard, the parameter estimates in the proportional hazards model were as follows:

Covariate	Parameter estimate	Parameter
Sex	β_1	+0.40
Cigarette smoking	β_2	+0.75
Alcohol consumption	β_3	-0.20
	β_0	-5.00
	c	+1.10

Table 2

- Calculate the estimated hazard for a female cigarette smoker aged 55 years who does not consume alcohol. (2 marks)
- Show that, according to this model, a cigarette smoker at any age has a risk of death roughly equal to that of a non-smoker aged eight years older (6 marks)

QUESTION FOUR (20 MARKS)

- a) A bank account pays 10% effective annual interest rate over 5 years. Find the equivalent:
- (i) simple annual interest rate (1 mark)
 - (ii) effective monthly interest rate (1 mark)
 - (iii) effective biennial interest rate (1 mark)
 - (iv) effective annual discount rate (2 marks)
- b) Express 5% p.a effective interest as a nominal annual interest rate convertible quarterly. (4 marks)
- c) The force of interest $\delta(t)$ is a function of time and at any time t , measured in years, is given by formula. $\delta(t) = 0.05 + 0.001t$, for $0 \leq t \leq 20$ and $\delta(t) = 0.05$, for $t > 20$.
- (i) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time t . (2 marks)
 - (ii) Calculate the present value of £100 due at the end of 25 years. (4 marks)
 - (iii) Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii). (5 marks)

QUESTION FIVE (20 MARKS)

- a) Calculate the following probabilities on the basis of ELT15 (males)
- i. ${}_{40}q_{25}$ (2 marks)
 - ii. The probability that (50) fails to reach age (70) (2 marks)
 - iii. The probability that (60) dies within the first five years after retiring at age (65). (2 marks)

- b) A life office issued 750 identical 25-year temporary assurance policies to lives aged 30 exact each with a sum assured of £75,000 payable at the end of year of death. Premiums are payable annually in advance for 20 years or until earlier death.
- i) Show that the annual net premium for each policy is approximately equal to £104 using the basis given below. (6 marks)
- ii) Calculate the net premium reserve per policy at the start and at the end of the 20th year of the policy. (8 marks)

Basis:

- Mortality: AM92
- Ultimate Interest: 4% per annum