



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (BIOLOGY)

SMA 101: MATHEMATICS FOR SCIENCE II

DATE: 9/8/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS:

Answer Question **ONE** which is compulsory and any other **TWO** Questions

QUESTION ONE (30 MARKS)

- a) At what values of x is the function $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ continuous? (3 marks)
- b) Determine the area of the region in the first quadrant that is bounded by $y = \sqrt{x}$, x -axis and the line $y = x - 2$ (4 marks)
- c) State the *L'Hôpital's Rule* and hence evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - (x/2)}{x^2}$ (4 marks)
- d) Determine $\frac{dy}{dx}$ given that
- $y = x^x$
 - $y = (x^2 + 3x)^7$ (6 marks)
- e) i Given that the domain of the function $y = \sqrt{5-x}$ is $\{-2, -1, 3\}$ find its range and Co-domain
- ii Given $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{x}{1-x}$ Determine $(f \cdot g)^{-1}$ (5 marks)
- f) Find from first principle the derivative of $y = x \sin 3x$ (3 marks)

- g) An object is launched straight up into the air at an initial velocity of 64m/s. six metres off the ground. If its height (H) in metres at t seconds is given by the formula $H = 16t^2 - 64t + 6$. determine
- The maximum height reached
 - The time it hits the ground and the velocity at the time (5 marks)

QUESTION TWO (20 MARKS)

- a) Determine the values of the gradients of the tangents drawn to the circle $x^2 + y^2 - 3x + 4y = -1$ at $x = 1$ correct to two significant figures (6 marks)
- b) Determine the area under the graph of a nonnegative function $y = x$ the x -axis and the lines $x = 1$ and $x = 4$ (6 marks)
- c) Determine and identify the stationary values of the function $f(x, y) = x^3 - 6x^2 - 8y^2$ (8 marks)

QUESTION THREE (20 MARKS)

- a) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (2 marks)
- b) The region bounded by the curve $y = \sqrt{x}$, x -axis and the line $x = 4$ is revolved about the x -axis to generate a solid. Determine the volume of the solid so formed. (5 marks)
- c) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5\text{cm}^2 / \text{s}$. Find the rate of change of the radius when the area is 30cm^2 (5 marks)
- d) If $y = 3e^{2x} \cos(2x - 3)$, Verify that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (8 marks)

QUESTION FOUR (20 MARKS)

- a) A 2.6m ladder is placed against a wall such that its foot is one metre from the wall. If the top of the ladder is sliding down the wall at $0.2\text{m} / \text{s}$, determine the rate at which the ladder is moving away from the wall (4 marks)
- b) Find $\frac{dy}{dx}$ at $x = 3.1$ given that $y = \frac{2x^3}{\cosh 3x}$ (6 marks)

- c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through (1, 0) and has a stationary point at (1, 0). determine the value of α , β and the other turning points (10 marks)

QUESTION FIVE (20 MARKS)

- a) State the mean value theorem and hence determine the value of the constant “c” that satisfy the theorem in $f(x) = x^3 + 2x^2 - x$, $[-1, 2]$ (6 marks)
- b) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (7 marks)
- c) Evaluate
- i. $\int x e^x dx$
- ii. $\int_0^{\frac{\pi}{2}} 5 \sin 2x dx$ (7 marks)