MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SUPPLEMENTARY EXAMINATION FOR BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SMA 201: CALCULUS III

DATE: 27/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer QUESTION ONE and other TWO QUESTIONS

QUESTION 1: COMPULSORY (30 MARKS)

a.) Show that $\dfrac{\sqrt{x}+\sqrt{y}}{x+y}$ is a homogeneous function of degree $\dfrac{-1}{2}$ (2 marks)

b.) Expand $\sin x$ as an infinite series (4 marks)

c.) Verify Lagrange's Mean Value Theorem for the function $f(x)=e^{x}$ in $[0 \le x \le 1]$ (4 marks)

d.) If $u = u(x, y, z) = \left(x^2 + y^2 + z^2\right)^2$, find $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}$. (5 marks)

e.) Find $\lim\limits_{x\to 0}\dfrac{\ln x}{\csc x}$ (5 marks)

f.) Verify Rolle's Theorem for the function $f(x)=(x-2)^6(x-5)^{11}$ in the interval $[2 < x < 5]$. (5 marks)

g.) Using Green's Theorem, evaluate $\int_C \left[y(2xy-1)dx + x(2xy+1)dy\right]$ where C the circle is $x^2 + y^2 = 9$. (5 marks)

QUESTION 2 (20 MARKS)

a.) Determine $\lim_{x \to a} \frac{\ln(x-a)}{\ln(e^x - e^a)}$ (4 marks)

b.) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ (4 marks)

c.) Verify Euler's theorem on the function $f(x) = 2x^3 + 3x^2 y - 4y^3$ (6 marks)

d.) Find the minimum value of $x^2 + y^2 + z^2$ when $x + 2y + 4z = 21$ is the constraint (6 marks)

QUESTION 3 (20 MARKS)

a.) Evaluate the double integral $\iint \frac{x}{y} dxdy$ in the region R bounded by the lines $|x| \le 1, 1 \le x \le 2.$

(8 marks)

b.) Verify Stoke's Theorem for the vector function $F = (2x - y)i - yz^2 j - yzk$, When S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 9$ and C is its boundary (12 marks)

QUESTION 4 (20 MARKS)

a.) Verify Green's Theorem in the plane for the function $P(x, y) = xy + y^2$ and $Q(x, y) = x^2$

(5 marks)

b.) (i) Expand $f(x) = x^2, -\pi \le x\pi$ in a fourier series (5 marks)

(ii) Using your results prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$ (10 marks)

QUESTION 5 (20 MARKS)

a.) If $f(x) = x^3 - 6x + 11x - 6$ and the interval is $[0 \le x \le 4]$ find c of the Lagrange mean value Theorem (6 marks)

b.) Find the maximum or minimum values of $2(x^2 - y^2) - x^4 + y^4$ (14 marks)