MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

SCO 111: DIFFERENTIAL CALCULUS FOR COMPUTER SCIENCE

DATE: 10/5/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine $\frac{dy}{dx}$ given that

i) $y = x^x$ (2 marks)

ii) $y = (x^2 + 3x)^7$ (3 marks)

b) It is estimated that t years from now the population of a certain estate in the periphery a major city will be p thousand people, where $p(t) = 20 - \frac{7}{t+2}$. A study conducted by the city planning department indicates that the average amount of waste will be w parts per thousand when the population is p thousand, where $w(p) = 0.4\sqrt{p^2 + p + 21}$.

i) Express w as a function of time t. (2 marks)

ii) What happens to the amount of waste w in the long run (as $t \to \infty$)? (3 marks)

c) Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (4 marks)

d) Calculate the maxima and minima values of the function $y = x^3 - 4x^2 - 3x + 2$ and distinguish between them. (5 marks)

e) Find the equation of the line tangent to the graph of implicit function $y^3 + x^2 - 3xy = 0$ at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$ and passing through $y = 3$. (5 marks)

e) State the mean value theorem of differential calculus and hence determine the value of the constant "c" that satisfy the theorem in $f(x) = x^3 + 2x^2 - x$, [-1,2] (6 marks)

QUESTION FOUR (20 MARKS)

a) Using logarithmic differentiation, differentiate the following functions;

i) $xe^x \sin x$ (3 marks)

ii) $te^t \cos t$ (3 marks)

b) Determine $\frac{dy}{dx}$ given that $f(t) = \frac{t^2}{\sqrt{t^2 + 4}}$ (5 marks)

c) A particle moves along a straight line and its displacement x meters from the origin, after t seconds is given by $x = 4.9t^2$. find in terms of t

i) The average velocity over the interval t to $(t + \partial t)$ (5 marks)

ii) The instantaneous velocity. (4 marks)

QUESTION FIVE (20 MARKS)

a) State the L'Hospital rule, hence or otherwise evaluate

$$\lim_{x \to -3} \frac{x+3}{x^2 + 5x + 6}$$

(2 marks)

b) An object moves along a coordinate line, its position at each time $t \geq 0$ given by $f(t) = 3t^2 - 7t + 4$. Find the position, velocity and acceleration at time $t = 4$sec.

(3 marks)

c) Determine the gradient of the curve $x^2 + 2xy - 2y^2 + x = 2$ at the point (-4, 1)

(5 marks)

d) Obtain $\frac{df(x)}{dx}$ for $f(x) = \frac{\sin x + e^{2x}}{\sin x}$ (5 marks)

e) A farmer has 60m of fencing wire which he can put against an existing fence to form a rectangular enclosure for animals. What is the maxima area it can enclose? (5 marks)

QUESTION FOUR (20 MARKS)

a) A curve is given by the parametric equations $x = a\cos^3\theta$ and $y = a\sin^3\theta$. Show that

$\frac{dy}{dx} = -\tan\theta$ also find the value of $\frac{d^2y}{dx^2}$ where $\theta = \frac{\pi}{4}$. (10 marks)

b) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through $(1,0)$ and has a stationary point at $(1,0)$. Find the value of α , β and the other turning points and hence sketch the curve (10 marks)

QUESTION FIVE (20 MARKS)

a) Prove that $\frac{d}{dx}(\sin x) = \cos x$ from the first principal. (4 marks)

b) If $y = 3e^{2x}\cos(2x-3)$ verify that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (6 marks)

c) A cylinder is to be constructed so that the sum of height and base radius is 6cm. denoting base radius by rcm and volume Vcm^3. Show that $V = \pi\left(6r^2 - r^3\right)$. Hence show that the value of r which makes V a maxima. (10 marks)