MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF (ENVIRONMENTAL RESOURCE AND CONSERVATION)

BACHELOR OF (ENVIRONMENTAL SCIENCE)

ENS 137: MATHEMATICS FOR ENVIRONMENTAL SCIENCES

DATE: 8/5/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

1) **Answer Question ONE and any other TWO questions.**
2) **Mobile phones and any written material are prohibited in the examination room.**
3) **No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.**
4) **All answer booklets should be handed in at the end of the exam whether used or not.**
5) **Programmable calculators are prohibited**

QUESTION ONE (COMPULSORY) (30 MARKS

a) Show that $y = x^2$ is a solution of the equation $3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 10x + 6$ (3 marks)

b) Solve the equation $(x^3 + y^3)dx - 3x^2y^2dy = 0$ (5 marks)

c) Find $\int \left[\frac{x^2+5x+6}{(x+2)}\right] dx$ (4 marks)

d) Evaluate the area enclosed between the curve $y = x^2 - 5x + 4$ and the x-axis (5 marks)

e) Find the value of c such that $\begin{vmatrix} c & -3c \\ -1 & -c \end{vmatrix} = -2$ (4 marks)

f) Solve the following system using Cramer's rule. (5 marks)

$$-2x_1 + 3x_2 - x_3 = 1$$

$$x_1 + 2x_2 - x_3 = 4$$

$$-2x_1 - x_2 + x_3 = -3$$

g) Work out the derivative of the function below using product rule
$f(x) = (1 - 5x^2)(6x^2 - 4x + 1)$ (4 marks)

QUESTION TWO (20 MARKS)

a) If $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 2 & -1 \end{bmatrix}$. Find the matrix $C = 3A - 2B$ (2 marks)

b) Solve the following systems of simultaneous equations using Gauss-Jordan elimination method , (6 marks)

$$\begin{aligned} 2x_1 - 4x_2 + 6x_3 &= 20 \\ 3x_1 - 6x_2 + x_3 &= 22 \\ -2x_1 + 5x_2 - 2x_3 &= -18 \end{aligned}$$

c) (i) Differentiate between a scalar quantity and vector quantity (2 marks)

(ii) Find the cross product of $\vec{a}$ and $\vec{b}$ given that $\vec{a} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\vec{b} = \boldsymbol{j} + 3\boldsymbol{k}$ (3 marks)

d) Let $A = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{bmatrix}$

Find (i) $\det A$ (2 marks)

(ii) $Adj(A)$ (3 marks)

(iii) Hence find A^{-1} (2 marks)

QUESTION THREE (20 MARKS)

a) Use substitution method to find the integral of $\int (\tan x)^{10} sec^2 x \, dx$ (5 marks)

b) Use partial fractions to find the integral $\int \frac{x-11}{(x+3)(x-4)} dx$ (5 marks)

c) Work out $\int_2^3 (x^2 + 2x) dx$ (4 marks)

d) Find the area enclosed by the line $y = 4x$ and the curve $y = x^2$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Find the order and the degree of the following differential equation

$\left[\frac{d^2y}{dx^2}\right]^3 + 10\left[\frac{dy}{dx}\right]^8 + 9y = \cos x$ (2 marks)

b) Solve the equation $(x^2 - 3y^2)dx + 2xydy = 0$ (8 marks)

c) Show that the equation, $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$ is exact hence solve the equation. (8 marks)

d) Evaluate $\frac{\partial^2}{\partial x^2}(8x^2 + 9x^2 - 7y^2)$ (2 marks)

QUESTION FIVE (20 MARKS)

a) Given that $k(x) = \frac{2x^2+1}{x^2-1}$, find $k'(x)$. (4 marks)

b) Identify the stationary points on the curve $y = x^3 - 3x + 2$. For each point determine whether it is a maximum, minimum or a point of inflection. (6 marks)

c) A closed cylindrical metal tin is to have a capacity of 250π ml. If the area of the metal used is to be a minimum, what should the radius of the tin be? (7 marks)

d) The displacement, s meters covered by a moving particle after time t seconds is given by

$s = 2t^3 + 4t^2 - 8t + 3$. Find the velocity at $t = 2$. (3 marks)