



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 203 LINEAR ALGEBRA II

DATE: 22/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30MARKS)

- a) State the Cayley Hamilton theorem (3 marks)
- b) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and let $g(x) = x^2 - 5x - 2$, $f(x) = 2x^2 - 3x + 7$. Show that A is a root of $g(x)$ and not a root of $f(x)$ (4 marks)
- c) Express $(2, -5, 3)$ as a linear combination of $(2, -3, 2)$, $(2, -4, 1)$ and $(1, -5, 7)$ (4 marks)
- d) Let $f: V \rightarrow V$ be a linear mapping. Let A be the matrix relative to a basis $v_1, v_2, v_3, \dots, v_n$. Prove that the columns of A span $\text{im } f$ (4 marks)
- e) Define a linear mapping (3 marks)
- f) Let $v \in \mathbb{R}^4$; $w = \{(x, y, z, w) / x + y - z = 0\}$ by $f(x, y) = (x + y, 0)$. show that w is a subspace of $\mathbb{R}^4$ (5 marks)
- g) Prove that “any set of linearly independent vectors in V can be extended into a basis for V ” (4 marks)
- h) Let $V = P_3(x)$ and define $D: v \rightarrow v$ by $Df = \frac{df}{dx}$, D is linear. Determine
- i) Nullity (2 marks)

ii) Rank

(1 mark)

QUESTION TWO (20 MARKS)

- a) Define $f : \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$ by $f(x, y, z) = (x + y, z, -x)$. Let $S = \{(1, 1, 1), (1, 2, 1), (0, -1, 1)\}$ be an ordered basis for $\mathfrak{R}^3$. Determine the matrix of f relative to the basis S . (8 marks)
- b) If $f : v \rightarrow w$ (v, w vector spaces) prove that;
- i. $\ker f$ is a subspace of v (6 marks)
- ii. $\text{im } f$ is a subspace of w (6 marks)

QUESTION THREE (20 MARKS)

- a) Determine the minimal polynomial of the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix} \quad (7 \text{ marks})$$

- b) Let V be a subspace of R^4 spanned by $(1, -1, 0, 0)$ $(0, 0, 1, -1)$ $(0, 0, 3, 1)$ and let u be the subspace of R^4 spanned by $(1, 2, -2, 3)$ $(3, 0, 2, -1)$ $(2, 1, 0, 1)$. Determine a basis for;
- i) $U + V$ (6 marks)
- ii) $U \cap V$ (7 marks)

QUESTION FOUR (20 MARKS)

- a) Consider the matrix $A = \begin{pmatrix} -1 & -5 \\ 10 & 14 \end{pmatrix}$. Work out the matrices P and P^{-1} such that $P A P^{-1} = D$. Where D is a diagonal matrix. (6 marks)
- b) Prove the dimension's formula $\dim(u + v) = \dim u + \dim v - \dim(u \cap v)$ (7 marks)
- c) Calculate the eigen values and the eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} \quad (7 \text{ marks})$$

QUESTION FIVE (20MARKS)

- a) Let $f : \mathfrak{R}^n \rightarrow \mathfrak{R}^m$ have the matrix $A = \begin{pmatrix} 2 & 1 & -1 & 3 \\ 3 & 0 & 1 & 4 \\ 1 & 2 & -3 & 2 \end{pmatrix}$ with respect to standard basis.

Determine;

- i) The value of m and n (3 marks)
 - ii) The $\text{im } f$ and $\text{rank } f$ (5 marks)
 - iii) The $\ker f$ and $\text{nullity } f$ (5 marks)
- b) Let V be a vector space and let w be a fixed vector in V . Define $W = \{\text{set of all vectors in } V \text{ which are } \perp w\}$
- $v_1 \perp v_2 \Leftrightarrow v_1 \text{ is perpendicular to } v_2$.
- $W = \{v \in V / v \cdot w = 0\}$. Show that W is a subspace of V . (7 marks)