



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 301: ACTUARIAL MATHEMATICS II

DATE: 24/8/2022

TIME: 11.00-1.00 PM

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE: (30 MARKS)

a) Define the following actuarial notations

i. ${}_tq_{\overline{xy}}$ (2 marks)

ii. $K_{\overline{xy}}$ (2 marks)

b) Using the PMA92C20 table for both lives calculate:

i. $\mu_{65:60}$ (3 marks)

ii. ${}_5p_{65:60}$ (3 marks)

c) The in force expected cash flows for a 6-year policy issued to lives aged x is:

$(-131.53, -70.11, 25.00, -20.15, 55.74, 157.91)$

Assume non-unit reserves earn interest at 6% pa and that the probability of death during any year is 0.01.

Calculate the reserves required for the policy if the cash flow in Year 3 is 15.00 instead of 25. (5 marks)

d) Consider a person now aged exactly 25 whose annual salary rate is currently £9,192. Estimate his annual salary rate at exact age 53 (5 marks)

- e) The random variable T_{xy} represents the time to failure of the joint- life status (xy). (x) is subject to a constant force of mortality of 0.02 and (y) is subject to a constant force of mortality 0.03. (x) and (y) are independent with respect to mortality. Calculate the value of $E[T_{xy}]$ (5 marks)

- f) In a combined sickness and mortality table

$$K_{x+1} = 554,405$$

$$z_x = 1.129$$

$$D_x = 24,510$$

$$D_{x+1} = 23,425$$

- Estimate K_x . (5 marks)

QUESTION TWO (20 MARKS)

In a certain population, forces of decrement are assumed to be constant over individual years of age. The following independent forces of decrement will be assumed for this population between the exact ages of 50 and 52.

Age (x)	Force of decrement for the year of age due to mortality	Force of decrement from exact age x due to sickness
50	0.011	0.075
51	0.012	0.081

Table 1

Construct a double decrement table including the two decrements of mortality and sickness, for this population between exact ages 50 and 52,

Assuming a radix of $(al)_{50} = 100,000$

QUESTION THREE (20 MARKS)

A certain friendly society recruits only married men aged under 55. The society provides the following benefits:

- immediately on the death of the member at any age, £2,000
- during the first 10 years of membership, immediately on the death of the member's wife before her husband, £500,
- on survival of the member to age 65, an annuity of £10 per week for life, and

- During any spell of sickness of the member before age 65, £10 per week reducing to £6 per week after 3 months' sickness.

The basis for all calculations is:

Mortality of members: E.L.T. No.12 - Males

Mortality of wives: E.L.T. No. 12 - Males with an age-deduction of 5 years

Interest: 4% per annum

Sickness: Manchester Unity 1893-97 AHJ

Expenses: 5% of all contributions, including those waived during sickness

Wives of members are taken as being of the same age as their husbands; there is no benefit on the death of wives of marriages taking place after entry to the society, and the possibility of divorce is to be ignored. There is no waiting period and the off-period is as in the Tables provided. The possibility of withdrawal is ignored.

- Calculate the weekly contribution rate, waived during sickness and ceasing at age 65 or the previous death of the member, for an entrant aged 25. (17 marks)
 - Calculate the reserve for a member aged 35 who joined at age 25. (3 marks)
- Note Use Simpson's rule for approximate integration, i.e.

$$\int_a^b f(t)dt \cong \frac{b-a}{6} [f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)]$$

QUESTION FOUR (20 MARKS)

The actuary advising a pension scheme has decided that the independent mortality in the standard table for pension schemes (PEN) from page 142 of the Formulae and Tables for Actuarial Examinations is no longer appropriate for that pension scheme.

Calculate the revised row of the service table for age 61, assuming that the revised independent mortality rate at that age is 80% of the previous independent mortality rate.

QUESTION FIVE (20 MARKS)

A life insurance company issues a number of 3-year term assurance contracts to lives aged exactly 60. The sum assured under each contract is £200,000, payable at the end of the year of death. Premiums are payable annually in advance for the term of the policy, ceasing on earlier death.

The company carries out profit tests for these contracts using the following assumptions:

- Initial expenses: £200 plus 35% of the first year's premium
 - Renewal expenses: £25 plus 3% of the annual premium, incurred at the beginning of the second and subsequent years
 - Mortality: AM92 Ultimate
 - Investment return: 7% per annum
 - Risk discount rate: 15% per annum
 - Reserves: One year's office premium
- a) Show that the office premium, to the nearest pound, is £2,527, if the net present value of the profit is 25% of the office premium. (13 marks)
- b) Calculate the cash flows if the company held zero reserves throughout the contract, using the premium calculated in part (a). (5 marks)
- c) Explain why the company might not hold reserves for this contract and the impact on profit if they did not hold any reserves. (2 marks)