MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 201: CALCULUS III

DATE: 19/4/2023 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer Question One and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the following theorems
   i) Rolle's theorem (2 marks)
   ii) Mean value theorem (2 marks)

b) Distinguish between Taylor's and Maclaurin's series (4 marks)

c) Find the limit $\lim_{x \to -10} \frac{x^2-100}{x+10}$ using L'Hospital's rule and verify the results using an alternative method (3 marks)

d) Determine whether the limit below exists along the lines $y = 2x$ and $x = 0$ and if so find the value $\lim_{(x,y) \to (0,0)} \frac{x^4-y^4}{x^2+y^2}$ (5 marks)

e) Examine the validity of the hypothesis and conclusion of the specified theorem in each case below.
   i) $f(x) = x^3 - 3x^2 + 2x$ on $[0, 0.5]$ (Mean Value Theorem) (3 marks)
   ii) $f(x) = 2 - (x-1)^2$ on $[0,2]$ (Rolle's theorem) (3 marks)

f) Expand, if possible, $f(x) = e^{2x}$ in ascending powers of x up to the third term. (4 marks)

g) Obtain the first partial derivatives for

i) $f(x,y) = lnx^3y - e^{xy}$ (2 marks)

ii) $f(x,y) = -xy^2 + 3y + 2x^2$ (2 marks)

QUESTION TWO (20 MARKS)

a) Obtain the first four Taylor polynomials for $f(x) = \frac{1}{x}$ about $x = 1$ (5 marks)

b) Obtain formulae for the Taylor series for the $f(x) = e^{1-x}$centered at $x = 1$ as far as$(x-1)^3$and state the coefficient of $(x-1)^2$ (5 marks)

c) Determine the stationary values of $z = x^3 - 3x + xy^2$ and state their nature.(10 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\iint_R (x+y)dxdy$ where R is a rectangle $0 \le x \le 2,\ 0 \le y \le 4$ (5 marks)

b) Find the extreme points for $f(x,y) = 4x^3 - 3x^2y + 2xy^2 - 15y^3$.$f(x,y) = 4x^3 - 3x^2y + 2xy^2 - 15y^3$ (5 marks)

c) Determine that slope of a line tangent to the surface $4z = 3x^2 + 4y^2$and parallel to the $xz -$ plane at the same point and find the angle of inclination (6 marks)

d) Show that the equation has exactly one solution in the given interval $x^4 + 2x^3 - 2 = 0$ $[0,1]$ using Rolle's Theorem. (4 marks)

QUESTION FOUR (20 MARKS)

a) State the Green's theorem (2 marks)

b) Evaluate the line integral given below using Green's theorem $\oint_c y^2dx + x^2\,dy$ where c is the square with vertices (0,0), (2,0), (2,2), (0,2)oriented counterclockwise. (8 marks)

c) Find the stationary points of the function $z = x^2 + y^2$ subject to the constraints $x^2 + y^2 + 2x - 2y + 1 = 0$ (5 marks)

d) Evaluate the limit of $\lim_{(x,y)\to(2,1)} \frac{(x^2-2xy)}{(x^2-4y^2)}$ (5 marks)

QUESTION FIVE (20 MARKS)

a) State the L'Hospital's rule (2 marks)

b) Find the Maclaurin's series for $f(x) = \frac{1}{x-1}$ (6 marks)

c) A solid is enclosed by the planes $z = 0, y = 1, x = 0, x = 3$ and the surface $z = x + y^2$. Determine the volume of the solid so formed. (6 marks)

d) Evaluate the double internal $I = \iint_R x^2ydxdy$ where R is the triangular area bounded by the lines $x = 0, y = 0$ and $x + y = 1$ (6 marks)