



# MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR  
BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION  
TECHNOLOGY)

SPH 205: MATHEMATICAL PHYSICS

DATE: 31/8/2022

TIME: 11.00-1.00 PM

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**INSTRUCTIONS:**

Answer question **ONE** which is compulsory and any other **TWO**

**SECTION A (COMPULSORY)**

**QUESTION ONE (30 MARKS)**

- a) (i) Distinguish between dot and cross product of a vector (2 marks)
- (ii) Give examples of physical situations where each product may be used (2 marks)
- b) Vectors  $\mathbf{A}$  and  $\mathbf{B}$  are defined as  $\mathbf{A} = 4\mathbf{i} + 3\mathbf{j} + 12\mathbf{k}$  and  $\mathbf{B} = 8\mathbf{i} - 6\mathbf{j}$ . Determine
- (i) Unit vectors  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{B}}$  (4 marks)
- (ii) Directional cosines of  $\mathbf{A}$  (3 marks)
- (iii) Angle between  $\mathbf{A}$  and  $\mathbf{B}$  using the cross product (3 marks)
- c) For the vectors in (b), show that  $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$  (4 marks)
- d) A force is given by  $\mathbf{F} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ .
- (i) Calculate the work done by the force in the direction of  $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ . (2 marks)
- (ii) Angle between  $\mathbf{F}$  and  $\mathbf{r}$  using dot product (3 marks)
- e) Show that the points A(1, 2, 3), B(3, 8, 1) and C(7, 20, -3) are collinear (4 marks)
- f) Show that the vectors  $\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ ,  $\mathbf{B} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and  $\mathbf{C} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$  are coplaner vectors (4 marks)

## QUESTION TWO (20 MARKS)

- a) A particle moves in space such that at any time  $t$ , its position,  $\mathbf{r}$  is given by  $x = 2t + 3$ ,  $y = t^2 + 3t$  and  $z = t^3 + 2t^2$ . At the time  $t = 1$  second, determine the following along the path given by  $\mathbf{P} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$
- (i) Velocity (4 marks)
  - (ii) Acceleration (4 marks)
- b) The path described by a particle is given by  $x = 2u$ ,  $y = u^2 + 3$  and  $z = 2u^2 + 5$ . Determine the unit tangent vector at the point (2,4,7) (4 marks)
- c) A charged particle is moving at a velocity given by  $\mathbf{V} = 2u\mathbf{i} - 3u\mathbf{j} + (u - 2)\mathbf{k}$  within a magnetic field  $\mathbf{B} = 3u\mathbf{i} + u^2\mathbf{j} + (u + 2)\mathbf{k}$ . E
- (i) If the force  $\mathbf{F} = \mathbf{B} \times \mathbf{V}$ , determine  $\mathbf{F}$  at  $u = 1$  (4 marks)
  - (ii) Hence evaluate  $\int_0^2 (\mathbf{B} \times \mathbf{V}) du$  (4 marks)

## QUESTION THREE (20 MARKS)

The vector fields  $\mathbf{A} = x^2y\mathbf{i} + (xy + yz)\mathbf{j} + xz^2\mathbf{k}$  and  $\mathbf{B} = yz\mathbf{i} - 3xz\mathbf{j} + 2xy\mathbf{k}$  as well as a scalar field  $\phi = 3x^2y + xyz - 4y^2z^2 - 3$  exist within a certain region, determine the following parameters

- (a) Grad  $\phi$ , (3 marks)
- (b) Div  $\mathbf{A}$  (3 marks)
- (c) Grad  $(\mathbf{A} \cdot \mathbf{B})$  (4 marks)
- (d) Curl  $\mathbf{A}$  (4 marks)
- (e) Curl Div  $\mathbf{B}$  (6 marks)

## QUESTION FOUR (20 MARKS)

- a) Show that the
- (i) Equation  $x^2y'' + xy' + (x^2 - p^2)y = 0$  is a singular point of the Bessel function (2 marks)
  - (ii) Bessel function of order zero is given by  $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 6^2} + \dots$  (5 marks)
- b)
- (i) State the Greens' theorem (2 marks)
  - (ii) Use the Greens' theorem to evaluate the closed path integral  $\oint \{(x^2 + y^2)dx + (x + 2y)dy\}$  taken round the boundary curve given by  $y = 0$ ,  $0 \leq x \leq 2$ ;  $x^2 + y^2 = 4$ ,  $0 \leq x \leq 2$  and  $x = 0$ ,  $0 \leq y \leq 2$  (6 marks)
- c) Find the Laplace transform for the function,  $f(t) = \sin at$  (5 marks)

**QUESTION FIVE (20 MARKS)**

- a) Distinguish Hermite and Laguerre polynomials using their characteristic equations as well as solutions (4 marks)
- b) Determine the Legendre polynomials of the function  $f(x) = x$  (5 marks)
- c) A bar of length 2 m is fully insulated along its sides. It is initially at a uniform temperature of  $10^{\circ}\text{C}$  and at time  $t = 0$  the ends are plugged into ice then maintained at a temperature of  $0^{\circ}\text{C}$ . Using separation of variables, determine the expression for the temperature at a point P, a distance  $x$  from one end at any subsequent time  $t$  seconds. (11 marks)