MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SPH 301: QUANTUM MECHANICS 1

DATE: 23/4/2019 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

- The paper consists of **two** sections.
- Section **A** is **compulsory**.
- Answer any **two** questions from section **B**.

The following constants may be useful

electron mass m_e= 9.109x10^{-31}Kg, electron volt eV=1.602 x10^{-19}J, charge e= 1.602x10^{-19}C, Gas constant R= 8.314J/K.mol, Gravitational constant G= 6.672x10^{-11}N.m^2/kg, neutron mass m_n=1.674x10^{-27}Kg, Permeability of free space $\mu_o = 4\pi\ x\ 10^{-7} T.m/A$, Permetivity of free space $\epsilon_o = 8.854\ x\ 10^{-12} C^2/N.m^2$. Plank's Constant $h = 6.626\ x\ 10^{-34} J.s$, ħ=1.054 x$10^{-34} J.s$, Proton mass $m_p = 1.672\ x10^{-27} Kg$, Rydberg constant $R_H = 1.097\ X\ 10^7 m^{-1}$, speed od light in a vacuum c = 2.997 x $\frac{10^8 m}{s}$,

Answer question **ONE** and any other **TWO** questions in section two

SECTION A

QUESTION ONE (30 MARKS)

a) A particle in a box of width L is described by the wave function $\psi(x) = 4 \sin 2\pi x/L$ find the value of amplitude and thus give the normalized wave function of the particle. (6 marks)

b) A wave function of a particle is described by $\psi = A\, e^{+i\omega(t+x/v)}$, express ψ in terms of sine and cosine. (4 marks)

c) Given that $\psi = b\, x^6 y^2$, find $\frac{\partial^2 \psi}{\partial x^2}$ and $\frac{\partial^2 \psi}{\partial y^2}$ (4 marks)

d) Derive the expression $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ which describes simple Harmonic motion.(4 marks)

e) Find the value of the normalization constant A for the wave function $\psi = A\, xe^{-x^2/2}$

f) An eigen function of the operator $\frac{d^2}{dx^2}$ is sin nx, where n =1, 2, 3, ... Find the corresponding eigen values. (4 marks)

g) Find the probability that a particle trapped in a box L wide can be found between 0.3L and 0.6L for the first 3 states. (4 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) The wave function $y = A \cos \omega(t - x/v)$, describes a wave that moves in the $+x$ direction along a stretched string.

i. Write down the its wave equation (2 marks)

ii. Show that this formula is a solution to the wave equation. (8 marks)

b) Given $\psi = Ae^{-(i/\hbar)(Et-px)}$ derive the time dependent form of the Schrödinger equation.

(10 marks)

QUESTION THREE (20 MARKS)

a) Derive the expression $\psi = Ae^{-(i/\hbar)(Et-px)}$ of a wave equivalent of an unrestricted particle of total energy E and momentum p moving in the $+x$ direction. (8 marks)

b) A beam of electrons is incident on a screen through two slits as shown in **Figure 1.** Redraw the diagram and

i. On line 'a' and 'b' show intensity pattern of electrons if only slit S1 is open only and when only slit S2 is open respectively. (2 marks)

ii. On lines 'c' and'd' draw expected intensity and actual intensity if both slits are open respectively. (2 marks)

iii. For each case in i) above represent the information using $|\psi|^2$ the probability of finding an electron. (4 marks)

iv. Hence show that Wave functions add, not probabilities given that P_1, P_2, ψ_1 and ψ_1 are probability densities and wave functions for the electrons corresponding to slit S1 and S2 respectively. (4 marks)

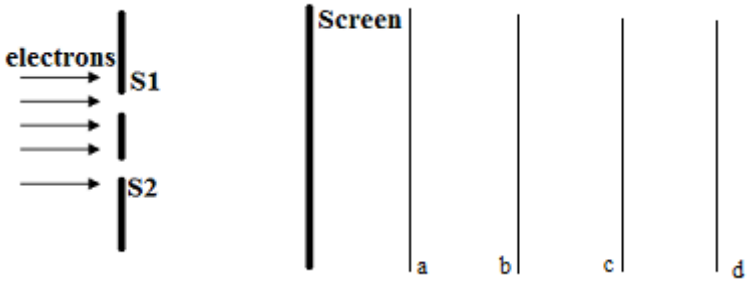

Figure 1

QUESTION FOUR (20 MARKS)

a) Find the expectation values $<x>$ and $<x^2>$ for the first two states of a harmonic oscillator. (12 marks)

b) Show that the harmonic-oscillator wave function ψ is a solution of Schrödinger's equation. (8 marks)

QUESTION FIVE (20 MARKS)

Electrons with energies of 0.50 eV and 3.0 eV are incident on a barrier 10.0 eV high and 0.80 nm wide.

a) Find their respective transmission probabilities. (12 marks)

b) How are these affected if the barrier is doubled in width? (8 marks)