



MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: 25/8/2022

TIME: 2.00-4. 00 PM

INSTRUCTIONS

Answer **question One (Compulsory)** and any other **Two questions**

QUESTION ONE (30 MARKS)

- a) i) Define a periodic function (2 marks)
- ii) Give one example of a periodic function (2 marks)
- iii) Define the Laplace transform of $f(x)$ (2 marks)
- b) Determine the Laplace transforms of:
- i. e^{at} (3 marks)
- ii. $\cos at$ (3 marks)
- c) Determine;
- i. $L^{-1}\left\{\frac{4s-5}{s^2-s-2}\right\}$ (5 marks)
- ii. $L^{-1}\left\{\frac{2s-3}{s(s-3)}\right\}$ (5 marks)
- iii. $L^{-1}\left\{\frac{1}{s^2+9}\right\}$ (5 marks)
- d) Determine the Laplace transforms of
- i. $L\{1 + 2t - \frac{1}{3}t^4\}$ (2 marks)
- ii. $L\{5e^{2t}-3e^{-t}\}$ (2 marks)
- iii. $\sin^2 t$ (2 marks)
- iv. $\text{Cos h } 3x$ (2 marks)

QUESTION TWO (20 MARKS)

- a) Obtain the Fourier Series for the periodic function $f(x)$ defined as

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases} \quad (16 \text{ marks})$$

The function is periodic outside this range with period 2π

- b) If in the F(s) above $k=1$, deduce the series for $\pi/4$ at the point $x=\pi/2$ (4 marks)

QUESTION THREE (20 MARKS)

Use the Laplace Transform to solve the differential equations

a) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$ given that $x=0, y=3, \frac{dy}{dx} = 7$ (10 marks)

b) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 9$ given that $x=0, y=0, y' = 0$ (10 marks)

QUESTION FOUR (20 MARKS)

Determine the Fourier Series for the full wave rectified sine wave

$$i = 5 \sin \frac{\theta}{2}$$

Commented [SK1]:

QUESTION FIVE (20 MARKS)

Consider Bessel's equation

$$z^2 y''(z) + zy'(z) + (z^2 - v^2)y = 0$$

Where $v \geq 0$, Find the Frobenius Solution that is asymptotic to t^v as $t \rightarrow 0$. by multiplying this solution by a constant find the solution

$$J_v(z) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k! \Gamma(k+v+1)} \left(\frac{z}{2}\right)^{2k+v}$$