



# MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

**HPH-100: APPLIED TECHNICAL MATHEMATICS**

**SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION**

## QUESTION ONE (30 MARKS)

a) Define the following terms: [4 Mks]

i) Set

ii) Define cardinality of a set and give its symbolic notation

b) Evaluate the following expressions.

i)

$$\frac{(3+9 \times 6) \div 3 - 2 \div 2}{3 \times 6 + (4-9) - 3^2 + 5}$$
 [3 Mks]

ii)

$$\frac{1}{3} \text{ of } \left( 5\frac{1}{2} - 3\frac{3}{4} \right) + 3\frac{1}{5} \div \frac{4}{5} - \frac{1}{2}$$
 [4 Mks]

c) The 1<sup>st</sup> term of a geometric progression is 4 and the 6<sup>th</sup> term is 128. Determine the 8<sup>th</sup> and 11<sup>th</sup> terms. [4 Mks]

d) Expand  $(x+2)^5$  using binomial theorem expansion [3 Mks]

e) Identify the degree, leading term, and leading coefficient of the polynomial. [3 Mks]

$$4x^2 - x^6 + 2x - 6$$

- f) Write the following expressions in the form  $a + bi$ . [3 Mks]

$$(-3 + 2i)(4 - 4i)$$

- g) Find the derivative of the following function [3 Mks]

$$y = \frac{1}{x} + \sqrt{x^3}$$

- h) Evaluate the following integral [3 Mks]

$$\int \left( \frac{1}{x^2} + x \right) dx$$

### SECTION B: ANSWER ANY TWO QUESTIONS IN THIS SECTION

#### QUESTION TWO (20 MARKS)

- a) Define the term mathematical statement as used in logic and true statements [2 Mks]

- b) Evaluate the following expressions. [5 Mks]

$$\frac{1 + \sqrt{25} + 3 \times 2 - 8 \div 2}{3 \times 4 - \sqrt{(3^2 + 4^2)} + 1} - \frac{(4 \times 2 + 7 \times 2) \div 11}{\sqrt{9} + 12 \div 2 - 2^3}$$

- c) A man gets employment of \$ 750 per month with an annual increment of \$50. What does he earn in 10<sup>th</sup> year? [3 Mks]

- d) Find the derivative of  $y = (x^2 + 1)(x^3 + 3)$  [5 Mks]

- e) Consider the following universal set  $U$  defined as  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Consider other three sets defined as

$$A = \{1, 3, 4, 5\}$$

$$B = \{4, 5, 6, 2\}$$

$$C = \{1, 5, 7, 10\}$$

Find the following: [5 Mks]

(i)  $(A \cap B)^c$

(ii)  $A \cap (B \cup C)$

### QUESTION THREE (20 MARKS)

- a) Differentiate between tautology and contradiction [4 Mks]
- b) Express  $Z = \left( \frac{3-i}{3+2i} + \frac{2+i}{3-2i} \right)$  in the standard form  $x + yi$  [5 Mks]
- c) Find the derivative of  $y = (3x^2 + 2)^4$  [4 Mks]
- d) In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea? [3 Mks]
- e) Write the truth table for the exclusive disjunction of P and Q [4 Mks]

### QUESTION FOUR (20 MARKS)

- a) Given the following series 3, 9, 27, . . . what is the  $n^{\text{th}}$  of the series which gives a value of 59049? [3 Mks]
- b) Solve the simultaneous equations by substitution method [3 Mks]
- $$12x - 3y = 6$$
- $$-7x + 3y = -1$$
- c) In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee? [3 Mks]
- d) Let  $f, g : \mathfrak{R} \rightarrow \mathfrak{R}$  is defined by  $f(x) = 2x - 3$  and  $g(x) = \frac{x+1}{5}$  respectively. Find  $f^{-1}$  and  $g^{-1}$  [2 Mks]
- e) Given that  $f(x) = 2x + 1$  and  $g(x) = x^2 + 2x - 3$  Find: [3 Mks]
- i)  $(f \circ g)(x)$
- ii)  $g(-6)$
- f) Let P be the proposition “Today is Saturday” and Q be “I will go to Nairobi”. Write the following propositions symbolically:
- i) Today is Saturday or I will go to Nairobi but not both [1 Mk]
- ii) If and only if today is not Saturday, then I will go to Nairobi. [1 Mk]

g) Construct the truth table for  $\neg (P \vee Q)$

[4 Mks]

**QUESTION FIVE (20 MARKS)**

a) State any three logical operators/ connectors and write their symbols as discussed in logic

[3 Mks]

b) Consider the following propositions:

P: Mathematicians are generous.

Q: Spiders hate algebra.

Write the compound propositions symbolized by

[4 Mks]

i.  $\sim P \Rightarrow \sim Q$

ii.  $\sim P \Leftrightarrow \sim Q$

c) Given that  $z = 3 + 4i$  and  $w = 12 + 5i$ , write down the moduli and arguments of

[4 Mks]

i)  $z$

ii)  $zw$

d) Let  $S = \{a, b, c\}$ . Find all the subsets of  $S$ .

[4 Mks]

e) Determine whether or not each of the following relations is a function

[3 Mks]

i)  $y = 3x + 2$

ii)  $y < x + 4$

iii)  $y = 2x^2 + 1$

f) For the following function, determine whether it is one-to-one or onto  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  defined by

$f(x) = 2x + 1$

[2 Mks]