Machakos University College

(A Constituent College of Kenyatta University)

University Examinations 2015/2016

DEPARTMENT OF MATHEMATICS AND STATISTICS

Bachelor of Science in Statistics and Programming

Second Year First Semester

SST 201 **Operations Research I**

Date: 16/08/2016 Time: 8:30 - 11:30 am

Instructions to the Candidate:

1. *Answer **Question 1** and any other **two** questions.*
2. *You must have the following items for this paper:*
   - *Scientific Calculator;*
   - *Graph paper.*

1. (a) (i) Outline **four** conditions which must be satisfied for a problem to be solved using linear programming technique. (4 *marks*)

(ii) Outline any **four** sources of constraints in an optimisation problem involving linear programming. (2 *marks*)

(b) Explain each of the following types of variables as used in linear programming:

(i) Slack variable;

(ii) Surplus variable. (4 *marks*)

(c) A factory produces four types of electrical cables – A, B, C and D. The factory wishes to maximise contribution to revenue. The sale prices per metre of the cables are 40, 32, 60 and 54 respectively. The factory employs 200 skilled workers, 150 unskilled workers, and 100 casual workers who work a 40 hour week. The time it takes to produce one metre of each type of cable is as shown in the table below:

Product		A	B	C	D
Hours	Skilled	5	3	2	6
	Unskilled	4	7	3	5
	Casual	7	6	4	8

Formulate a linear programming model for this problem. (8 *marks*)

(d) Given the linear program model below:

Minimise : $z = x_1 + 2x_2 + 3x_3$

Subject to : $3x_1 + 4x_3 \leq 5$

$5x_1 + x_2 + 6x_3 = 7$

$8x_1 + 9x_3 \geq 2$

With : $x_1 \geq 0, \quad x_2 \geq 0$

(i) Express the linear program model in standard form. (2 *marks*)

(ii) Generate an initial feasible solution for the linear programming model. (2 *marks*)

(e) (i) State the general form of an LP model in matrix form for a minimisation primal program and its corresponding symmetrical dual. (4 *marks*)

(ii) Given the linear programming model below:

Minimise : $z = 8x_1 + 12x_2 + 5x_3$

Subject to : $2x_1 + 3x_2 + 6x_3 \leq 20$

$6x_1 + 8x_2 + 5x_3 \leq 40$

$7x_1 + 3x_2 + 6x_3 \leq 50$

$x_1 + 2x_2 + 4x_3 \leq 30$

With : $x_1, \quad x_2, \quad x_3 \geq 0$

Determine its symmetrical dual program. (4 *marks*)

2. An oil refinery plant produces two oil products – diesel and petrol. The cost of production for diesel is Ksh 40 per litre while the cost of production for petrol is Ksh 50 per litre. The oil products are to be sold in different countries which require both products in specified ratios, but with minimum quantities per month to meet their domestic demand. The oil requirement for the various countries in million litres is as shown in the table below:

Country	A	B	C	D
Diesel	3	8	3	1
Petrol	7	9	3	1
Minimum demand	420	720	240	100

The Ministry of Energy has issued a directive that a minimum of 20 million litres of diesel must be produced per month in conformity with an industrialisation strategy.

(a) Formulate a linear programming model for the problem. (6 *marks*)

(b) Using the graphical method, determine the optimum monthly production plan which minimises the production cost for the refinery. (10 *marks*)

(c) Interpret the optimum solution obtained in (b) above. (4 *marks*)

3. A fruit farm which produces three types of fruit juices – mangoes, oranges and pineapples, has its weekly production plan modelled as an LP program as shown below.

$$\text{Maximise :} \quad z = 5x_1 + 8x_2 + 10x_3$$

$$\begin{aligned} \text{Subject to :} \quad & 2x_1 + 3x_2 + x_3 \leq 400 \\ & x_1 + x_3 \leq 150 \\ & 2x_1 + 4x_3 \leq 200 \\ & x_2 \leq 50 \end{aligned}$$

$$\text{With :} \quad x_1, \; x_2, \; x_3 \geq 0$$

(a) Using the Simplex method, determine the production plan which maximises contribution to revenue for the farm. (16 *marks*)

(b) Interpret the solution obtained in (a) above. (4 *marks*)

4. Below is a linear programming model. Use it to answer the questions that follow.

$$\text{Maximise :} \quad z = 30x_1 + 40x_2$$

$$\begin{aligned} \text{Subject to :} \quad & x_1 + x_2 \leq 50 \\ & x_1 + 2x_2 \leq 80 \\ & 2x_1 + x_2 \leq 90 \end{aligned}$$

$$\text{With :} \quad x_1 \geq 0, \; x_2 \geq 0$$

(a) Using the Simplex method, determine the optimum solution for the linear program problem. (10 *marks*)

(b) Determine the symmetrical dual of the LP programming model above. (4 *marks*)

(c) Using the relationship between a primal program and its symmetrical dual program, derive the optimum solution of the dual program from the optimum solution of the primal. (6 *marks*)

5. An oil distribution company is required to meet the demands 70, 50, 30, 20 in thousand litres at various destinations from supplies 60, 10, 100 in thousand litres from various sources. The transport cost per unit amount (thousand litres) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 2 & 3 & 11 & 7 \\ 1 & 0 & 6 & 1 \\ 5 & 8 & 15 & 9 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the oil at the cheapest cost possible. Using the northwest corner rule or the least cost method, do the following for the transportation problem:

(a) Derive the initial basic solution; (2 *marks*)

(b) Determine the optimal solution that minimizes the cost of transport; (16 *marks*)

(c) Interpret the optimum solution obtained in (b) above. (2 *marks*)