



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 465: MEASURE AND PROBABILITY

DATE: 20/1/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE

- a) Let Ω be a non-empty set, π be a δ -algebra on Ω and μ be a measure defined on π . Define;
- A measurable space
 - A measure space
 - A complete measure space
 - μ - null set
 - μ -almost everywhere
- (10 marks)
- b) Let $\{\Omega, \pi, P\}$ be a probability measure space and $A_j \in \pi$ for $j=1,2,3,\dots$. Show that; $P(A_1) \leq P(A_2)$, if $A_1 \subset A_2$
- (4 marks)
- c) Let Ω be a non-empty set. Show that the following are δ -fields.
- $\{\Omega, \Phi\}$
 - Power set, $P\{\Omega\}$
 - $\pi_1 \cap \pi_2$ where π_1 and π_2 are δ -fields defined on Ω .
- (3 marks)
(3 marks)
(4 marks)
- d) Let $\{\Omega, \pi\}$ be a measurable space. Suppose $A_1, A_2 \in \pi$ and μ is a measure on π . Show that $\mu(A_1 \cup A_2) \leq \mu(A_1) + \mu(A_2)$.
- (5 marks)

QUESTION TWO

- a) State the axioms of ;
- a measure, μ
 - a probability measure, $\mathbf{P}$ (6 marks)
- b) Let x_n be a sequence of random variables. State and prove the Chebychevs inequality. (6 marks)
- c) Define;
- Convergence in probability
 - Convergence with probability 1
 - Convergence in mean square
 - Convergence in distribution (8 marks)

QUESTION THREE

- a) Let $\{ \Omega, \pi, \mathbf{P} \}$ be a probability measure and $\mathbf{P}_B$ be defined by;
- $$\mathbf{P}_B(A) = \mathbf{P}(A/B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} ; \mathbf{P}(B) > 0.$$
- Show that $\mathbf{P}_B$ is a probability measure. (7 marks)
- b) Show that an indicator function is a measure function (3 marks)
- d) Let $\Omega = \{2\}$.
- Determine the power set, $\mathbf{P}\{ \Omega \}$ (1 mark)
 - Show that $\mathbf{P}\{ \Omega \}$ obtained in (a) above is a δ -algebra. (4 marks)
- e) Let $x_1, x_2, \dots, x_n$ be a sequence of random variables. Show that convergence in mean square implies convergence in probability. (5 marks)

QUESTION FOUR

- a) State the following theorems;
- Radon-Nikodym theorem (3 marks)
 - Fubini's theorem (5 marks)
- b) State the axioms of a δ -field and hence show that a δ -algebra is a δ -field. (10 marks)
- c) Define a Stieltjes measure. (2 marks)

QUESTION FIVE

- a) State and prove the continuity theorem of a measure, μ (10 marks)
- b) Define the power set of a set, $\mathbf{P}\{ \Omega \}$ and hence or otherwise
- determine the power set of a set $\{1,2\}$.
 - Determine whether the power set in (i) above is a δ -algebra. (10 marks)