



# MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 335: ODE I

DATE: 22/1/2021

TIME: 2.00-4.00 PM

## INSTRUCTIONS TO CANDIDATES

Attempt question *one* (*compulsory*) and any other *two questions*.

### QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Consider the differential equation  $\sqrt{3 + \left(\frac{dy}{dx}\right)^2} = 2x$ 
  - i.) State the order of the differential equation (2 marks)
  - ii.) State the degree of the differential equation (2 marks)
- b) Consider the differential equation  $(x^3 + y^3)dx - 3xy^2 dy = 0$ 
  - i) Show that it is homogeneous (3 marks)
  - ii) Use the substitution  $y = vx$  to transform the equation into an equation containing  $v$  and  $x$  only (2 marks)
- c) The scientific law states that the acceleration of a particle during upward motion under gravity is  $-g$ . write down this law as a differential equation (3 marks)
- d) Solve the differential equation  $\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y + 5}$ , given that it is non-homogeneous. (4 marks)
- e) Solve the Bernoulli's equation  $\frac{dy}{dx} + y = xy^3$  (4 marks)
- f) Determine the particular integral for the equation  $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$  (5 marks)

- g) Solve the linear equation  $\frac{d^4 y}{dx^4} + 2\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} = \sin 3x$  (5 marks)

### QUESTION TWO (20 MARKS)

- a) Solve the differential equation  $\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - \frac{dy}{dx} = e^{3x}$  (6 marks)
- b) Solve the differential equation  $x\frac{d^2 y}{dx^2} - \frac{dy}{dx} + 4x^3 y = x^5$  (7 marks)
- c) Solve the differential equation  $(D - 4)y = e^{3x} x^2$  (7 marks)

### QUESTION THREE (20 MARKS)

- a) Show that  $\frac{xy}{x^2 + y^2}$  is a homogeneous function in  $x$  and  $y$  only (5 marks)
- b) Solve the differential equation  $(3x + 2y - 5)dx + (2x + 3y - 5)dy = 0$  (8 marks)
- c) Solve the equation  $(D - 1)(D + 3)(D - 5)y = \cosh x$  (7 marks)

### QUESTION FOUR (20 MARKS)

Consider the differential equation  $(4x + 3y^2)dx + 2xdy = 0$

- a) Show that this equation is not exact (4 marks)
- b) Calculate an integrating factor of the form  $x^n$  where  $n$  is a positive integer (3 marks)
- c) Using the integrating factor in (b) above solve the differential equation (13 marks)

### QUESTION FIVE (20 MARKS)

- a) Determine the differential equation whose solution is  $y = A \cos 8x + B \sin 8x$  (4 marks)
- b) Solve the differential equation  $\frac{dy}{dx} = \sin x$  (3 marks)
- c) Solve  $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$  (5 marks)
- d) Solve  $(2D^2 + 2D + 3)y = x^2$  (8 marks)