



**MACHAKOS UNIVERSITY**  
**SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF**  
**SCIENCE IN COMPUTER SCIENCE**  
**SCO 111: DIFFERENTIAL CALCULUS FOR COMPUTER SCIENCE**

**INSTRUCTION:** Answer Question *ONE* which is compulsory and any other *TWO* Questions  
**QUESTION ONE (30 MARKS)**

a.) Determine the  $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 2}{10x^2 - x + 100}$  (2 marks)

b.) Differentiate  $x = t^3 + t^2, y = t^2 + t$  (3 marks)

c.) State the *L'Hôpital's Rule*, hence or otherwise evaluate

$$\lim_{x \rightarrow -3} \frac{x+3}{x^2+5x+6}$$
 (3 marks)

d.) Determine  $\frac{dy}{dx}$  given that  $y = \frac{\sin x}{x}$  (3 Marks)

e.) Find the derivative of the following function from the first principles.

$$f(x) = x^2$$
 (3 Marks)

f.) If  $y = \frac{u^2 - 1}{u^2 + 1}$  and  $u = \sqrt[3]{x^2 + 2}$  find  $\frac{dy}{dx}$ , (4 Marks)

g.) Obtain  $\frac{df(x)}{dx}$  for  $f(x) = \frac{\sin x + e^{2x}}{\sin x}$  (4 marks)

h.) Determine if the following function is continuous.  $g(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$  (4 Marks)

i.) Write down the derivative of  $\ln\left(\frac{1}{\sqrt{x}}\right)$  (4 marks)

### **QUESTION TWO (20 MARKS)**

- a.) Determine  $\frac{dy}{dx}$  given that  $x^2 + 2xy - 2y^2 + x = 2$ , at the point  $(-4, 1)$  (4 Marks)
- b.) Differentiate  $y = 3x^2 + \cos 2x$  from the first principles. (5 Marks)
- c.) Find the derivative the following functions  $y = x^2 - 4x$  and  $x = \sqrt{2t^2 + 1}$ , at  $t = \sqrt{2}$  using chain rule. (5 Marks)
- d.) Find the gradient of the curve  $x = \frac{t}{1+t}$ , and  $y = \frac{t^3}{1+t}$  at the point  $\left(\frac{1}{2}, \frac{1}{2}\right)$  (6 Marks)

### **QUESTION THREE (20 MARKS)**

- a.) Using logarithmic differentiation, differentiate the following functions;

- i.  $xe^x \sin x$  (3 Marks)
- ii.  $te^t \cos t$  (3 Marks)

- b.) Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$  (4 Marks)

- c.) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of  $5\text{cm}^2 / \text{s}$ . Find the rate of change of the radius when the area is  $30\text{cm}^2$  (5 Marks)

- d.) Evaluate the derivative of the function given as  $x^{\cos x}$  with respect to  $x$ , with respect to  $x$ , (5 marks)

### **QUESTION FOUR (20 MARKS)**

- a.) Differentiate  $y = (x^2 + 1)^3 (x^3 + 1)^2$  (5 Marks)
- b.) If  $x = t^3, y = t^2 - t$  find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  (5 Marks)
- c.) Find the maximum and minimum points of the curve  $y = x^2(x+1) = x^3 + x^2$  (10 Marks)

### **QUESTION FIVE (20 MARKS)**

- a.) Find  $\frac{dy}{dx}$  given that  $x^2 + 2xy - 2y^2 + x = 2$  at the point  $(-4, 1)$  (4 marks)

b.) Find the points on  $y = x^4 + 4x^3 - 6$  at which the gradient is zero and determine the nature of the points (8 Marks)

c.) The parametric equations of a curve are  $x = e^t$ ,  $y = \sin t$ , find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  as functions of  $t$ .

Hence show that  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ . (8 Marks)