MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

APRIL SESSION EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 336: ORDINARY DIFFERENTIAL EQUATION II

DATE: SCHOOL BASED **TIME:**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Reduce the fourth order equation $\frac{d^4y}{dx^4} - 5\frac{d^3y}{dx^3} + 7\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 6x = e^x$ to a system of four linear equations (3 marks)

b) Solve the total differential equation $yzdx - z^2dy - xydz = 0$ (5 marks)

c) State the existence and uniqueness theorem for the nth order linear differential equation. (3 marks)

d) Solve the non linear differential equation $x^2y\frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y\right)^2 - 3y^2 = 0$ (5 marks)

e) Show that the equation $2x^2y'' - xy' + (1 - x^2)y = 0$ has a regular singular point at $x = 0$ (4 marks)

f) i.) Define the wronskian of a set of functions $f = \{f_1\ ,\ f_2, f_3, ...f_n\}$ (3 marks)

ii.) Show that the wronskian of the functions $2e^{3x}$, e^{-x} , *and* e^{2x} is non-zero (4 marks)

g) determine the first integral of the differential equation $\frac{dy}{dx}\frac{d^2y}{dx^2} + x^2y\frac{dy}{dx} = xy^2$ (3 marks)

QUESTION TWO (20 MARKS)

a) prove that the Legendre polynomial of order 3 is given by $p_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$ (5 marks)

b) prove that the Bessel's function of order zero is given by

$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \ldots$ (5 marks)

c) the Rodriquez formula for Legendre's polynomial of order n is given as

$p_n(x) = \frac{1}{2^n n!} \frac{d}{dx^n}\left(x^2 - 1\right)^n$. determine the Legendre's polynomial for $p_4(x)$ (5 marks)

d) prove that $J_{-p}(x) = (-1)^p J_p(x)$ (4 marks)

QUESTION THREE (20 MARKS)

Consider the system of differential equation

$$\frac{dx}{dt} = x + e^t$$
$$\frac{dy}{dt} = -2x + 3y + 1$$

a) write the system in matrix form $x' = Ax + B$ (2 marks)

b) calculate the Eigen values of A (3 marks)

c) determine the Eigen vectors corresponding to each of the Eigen values (4 marks)

d) evaluate the general solution of $x' = Ax + B$ (11 marks)

QUESTION FOUR (20 MARKS)

a.) Given the differential equation $y'' - y = x$. Determine its power series solution at $x = 0$ (10 marks)

b.) Solve the second order nonlinear differential equation $\frac{d^2y}{dx^2}\sin\frac{dy}{dx} = \sin x$ which satisfies the condition $y(1) = 2 \quad and \quad \frac{dy}{dx} = 1$ by reducing the order. (10 marks)

QUESTION FIVE (20 MARKS)

Consider the system of differential equations below

$$\frac{dx}{dt} = -2x + 2x^2$$
$$\frac{dy}{dt} = -3x + y + 3x^2$$

Determine;

a) The critical points (4 marks)
b) The nature of the critical points (3 marks)
c) Linearize the system of the differential equation (8 marks)
d) Solve the system for x *and* y (5 marks)