



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SEMESTER EXAMINATION FOR
BACHELOR OF BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF SCIENCE IN STATISTICS AND MATHEMATICS

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SUPPLEMENTARY/SPECIAL EXAMINATION

SMA 322: METHODS OF APPLIED MATHEMATICS 1

DATE:

TIME:

INSTRUCTIONS

Attempt Question One And Any Other Two Questions.

QUESTION ONE (30 MARKS)

- a) Determine the inverse Laplace transform $L^{-1} \left\{ \frac{7s+13}{s(s^2+4s+13)} \right\}$ (6 marks)
- b) Find the orthogonal transformation which transforms the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to the canonical form. (6 marks)
- c) Deduce the Fourier series for the function $f(\theta) = \theta^2$ in the range 0 to 2π . (6 marks)
- d) Find the Laplace transform of $3e^{\frac{-x}{2}} \sin^2 x$. (6 marks)
- e) Determine the poles and zeros of the function $F(s) = \frac{(s+3)(s-2)}{(s+4)(s^2+2s+2)}$ and plot them on a pole-zero map. (6 marks)

QUESTION TWO (20 MARKS)

- a) Consider a stretched string of length 50cm which is set oscillating by displacing its mid-point a distance of 2cm from its rest position and releasing it with zero velocity. Solve the wave equation $u''(x) = c^{-2}u''(t)$ (10 marks)
- b) A metal bar, insulated along its sides, is 1m long. It is initially at room temperature of 15°C and at a time $t = 0$, the ends are placed at 0°C . Find an expression for the temperature at a point P at a distance x m from one end at any time t seconds after $t = 0$ using the heat conduction equation $u''(x) = c^{-2}u'(t)$. (10 marks)

QUESTION THREE (20 MARKS)

- a) Solve $u_{xx} + u_{yy} = 0$ by the separation of variables method (consider the case of the constant being zero). (10 marks)
- b) Define a singular Sturm-Liouville system. Give 4 differential equations which fall in this category. (10 marks)

QUESTION FOUR (20 MARKS)

- a) Determine the Fourier series for the periodic function $f(x) = \begin{cases} -2, & \text{when } -\pi < x < 0 \\ +2, & \text{when } 0 < x < \pi \end{cases}$ which is periodic outside this range of period 2π . (10 marks)
- b) Determine the Fourier series for the function $f(t) = t$ in the range $t = 0$ to $t = 3$. (10 marks)

QUESTION FIVE (20 MARKS)

- a) State and prove the theorem on orthogonality. (10 marks)
- b) Define Orthogonality. Hence show that $f(x) = \sin 2x$ and $g(x) = \cos x$ are orthogonal on $(-\pi, \pi)$ with respect to the weight function $r(x) = 1$. (10 marks)