MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF MATHEMATICS & COMPUTER SCIENCE

SMA 300: REAL ANALYSIS I

DATE: 26/7/2019 **TIME: 8:30 – 10:30 AM**

NSTRUCTIONS TO CANDIDATES

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

QUESTION 1: 30 MARKS (COMPULSORY)

(a) The following are subsets of $\mathbb{R}$:

(i) $A = (-3.2\ ,\ 1]$ (ii) $B = (-1\ ,\ \infty)$ (iii) $C = \{x: x^2 > 4\}$.

For each set examine boudedness and determine the following if they exist:

$\boldsymbol{infimum}$, $\boldsymbol{supremum}$, $\boldsymbol{maximum}$ and $\boldsymbol{minimum}$ values (6marks)

(b) Given that a and b are rational with $b \neq 0$ and s is an irrational number such that $a - bs = t$.

(i) Define what is meant by a rational number . (1 mark)

(ii) Show that t is irrational. (3marks)

(iii)Using (ii) above show that $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is irrational (2 marks)

(c) Define the following

(i) Neighborhood of a point (2 marks)

(ii) Interior point of a set (2 marks)

(iii) Open set (2 marks)

(iv) Real numbers ($\mathbb{R}$) (1 mark)

(d) Find the largest $\boldsymbol{\varepsilon}$ such that S contains an ε- neighborhood of $\boldsymbol{x_0}$ in each case :

(i) $\boldsymbol{x_0} = \frac{3}{4}$, $S = [\ \frac{1}{2}\ ,1)$ (ii) $\boldsymbol{x_0} = 5$, $S = (-1,\ \infty)$ (4 marks)

(e) Show that $\lim\limits_{n \to \infty} \left(\frac{3n+2}{n+1}\right) = 3$ using any appropriate technique. (3marks)

(f) Let $A_n = \left(1 - \frac{1}{n}\ ,2 + \frac{1}{n}\right)$, $n \geq 1$. Find:

(i)$\bigcup_{n=1}^{\infty} A_n$ (ii) $\bigcap_{n=1}^{\infty} A_n$ (4 marks)

QUESTION 2 (20 MARKS)

(a) Define what is meant by a convergent sequence (3 marks)

(b) Show from first principles that the sequence:

$x_n = 2 + \frac{(-1)^n}{n^2}\ \forall n \in N$ Converges to 2 in R (4 marks)

(c) Show that a sequence $\{x_n\}$ of real numbers converges to x if and only if every Subsequence of $\{x_n\}$ converges to x . (5marks)

(d) (i) Show that $\sqrt{2}$ is irrational (4marks)

(ii) State the completeness axiom (3 marks)

(iii) Use a counter example to show the incompleteness of rational numbers. (2 marks)

QUESTION 3 (20 MARKS)

(a) Show that the union of an arbitrary collection of open sets is an open set. (4 marks)

(b) Show that a finite collection of open sets is an open set.
Exhibit an example to show that an arbitrary (infinitely many) collection of open sets is not necessarily open. (4 marks)

(c) Show that every convergent sequence of real numbers is bounded. (4 marks)

(d) Suppose that x_n and y_n are convergent sequences with
Lim $x_n = x$ and Lim $y_n = y$ show that:

(i) Lim $(x_n + y_n) = x + y$ (4 marks)

(ii) Lim $(x_n y_n) = xy$ (4 marks)

QUESTION 4 (20 MARKS)

(a) Give the definition of the concepts of limit superior and limit inferior of any sequence (x_n) of real numbers (4marks)

(b) Using (a) above determine the convergence or divergence of the following sequences:

(i) $(x_n) = (-1)^n \left[1 + \frac{1}{3n}\right]$ (4marks)

(ii) $(x_n) = 7 - \left[\frac{(-1)^n}{n^2}\right]$ (4mark)

(c) Explain what is meant by countable and uncountable subsets of R giving relevant examples of each. (4marks)

(d) Show that the set Z of integers is countable, hence conclude that the set of rational is also countable. (4marks)

QUESTION 5 (20 MARKS)

(a) Prove that the series $\sum_{n=1}^{\infty} \left[\frac{1}{n^p}\right]$ converges if $p < 1$ and diverges if $p \leq 1$ (5 marks)

(b) Determine whether each of the following series converges or diverges.
If it converges find the sum to infinity.

(i) $\sum_{n=o}^{\infty} 2\left(cos\,\frac{\pi}{3}\right)^n$ (4 marks)

(ii) $\sum_{n=0}^{\infty} \; 2^{2n}3^{1-n}$ (4 marks)

(c) Apply integral test on the series :

$\sum \frac{1}{n^2+1}$ (3marks)

(d) Apply the indicated test of convergence:

(i) $\sum \frac{\sqrt{n}}{n^2+1}$ (comparison test) (2marks)

(ii) $\sum \frac{1}{n^n}$ (Root test) (2marks)