MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

BACHELOR OF SCIENCE IN CIVIL ENGINEERING

ECU 107 ENGINEERING MATHEMATICS IV

DATE: 24/3/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (**compulsory**) and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) Differentiate the following with respect to x

i. $\sin(x + y) = \cos y$ (4 marks)

ii. $y = sec^2 x$ (3 marks)

iii. $y = ln|1 - x|$ (3 marks)

iv. $y = e^{2x} \tan 3x$ (4 marks)

b) Find $\frac{dy}{dx}$ given that $x = \theta + \sin\theta, y = 1 - \cos\theta$ (3 marks)

c) Using the first principle method, find the gradients of the following at the specified point.

i. $y = 3x - 2$ at $x = 0$ (3 marks)

ii. $y = \frac{1}{3x+1}$ at $x = 1$ (4 marks)

d) Evaluate the following limits

i. $\lim_{x\to 0} \frac{4x^2+5x-2}{2x^3+3x^2}$ (3 marks)

ii. $\lim_{x\to 1} \frac{x^2+x-2}{x-1}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Find $\frac{dy}{dx}$ given that $y = (\sin t)\ ln|t^2 + 1|$ and $x = \frac{e^{3t}}{\cos\ 2t}$ at $t = 0$ (6 marks)

b) Using the first principle method, show that the derivatives of the functions given below are as shown against each function

i. $y = e^{2x}$, $\frac{dy}{dx} = 2e^{2x}$ (5 marks)

ii. $y = \sqrt{x}$, $\frac{dy}{dx} = \frac{\sqrt{x}}{2x}$ (4 marks)

iii. Given that $y = \sin x$, show that $\frac{dy}{dx} = \cos x$ (5 marks)

QUESTION THREE (20 MARKS)

a) Differentiate the following with respect to x

i. $y = x^2 \sin^{-1} x$ (4 marks)

ii. $y^2 + 6x = x^3$ (3 marks)

iii. $3y^2 + 2y + xy = x^3$ (3 marks)

b) If $y = \sqrt{t} + 1$ and $x = \sqrt{t^2 + 1}$,find $\frac{dy}{dx}$ when $t = 3$ (4 marks)

c) By differentiating the equation $x^2 - xy - y^2 = 3$ implicitly, evaluate $\frac{dy}{dx}$ at the point $(1,1)$ (3 marks)

d) Discuss the continuity of $f(x) = \frac{x^2+x}{x}$ at $x = 0$ (3 marks)

QUESTION FOUR (20 MARKS)

a) i. Sketch the curve $y = x^2 - 3x$ (4 marks)

ii. Using the sketch in (i) above determine the area of the region between $y = x^2 - 3x$ and the x-axis from $x = 0$ to $x = 4$ (4 marks)

b) Evaluate the following antiderivatives

i. $\int (5 - \frac{1}{\sqrt{x}})\ dx$ (2 marks)

ii. $\int x(x^4 + 2)^2 dx$ (4 marks)

c) Find an algebraic expression for the difference quotient $\frac{f(1+\Delta x)-f(x)}{\Delta x}$ when

$f(x)=x^2-\frac{1}{x}$ .Simplify the expression as much as possible. Then determine what happens as $\Delta x \to 0$. What is the value of $f'(1)$ (3 marks)

d) Find all local maximum and minimum points for $f(x)=x^{\frac{2}{3}}$ (3 marks)

QUESTION FIVE (20 MARKS)

a) Sketch the curves $y = 5x - x^2 - 4$ and $x - y - 1 = 0$ (6 marks)

b) Using (a) above find the area between the two curves (5 marks)

c) Evaluate $\int \frac{3x^2-2x+1}{\sqrt{x}}\, dx$ (3 marks)

d) Find the gradients of the ellipse $x^2 - 3xy + 2y^2 - 2x = 4$ at the points where $y = -1$ (6 marks)