MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

EEE 402: COMPLEX ANALYSIS FOR ENGINEERING

DATE: 19/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTION:

1) **Answer Question ONE and any other TWO questions.**
2) **Mobile phones and any written material are prohibited in the examination room.**
3) **No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.**
4) **All answer booklets should be handed in at the end of the exam whether used or not.**
5) **Programmable calculators are prohibited**

QUESTION ONE (COMPULSURY) (30 MARKS)

(a) Define the following terms

(i) Analytic function (1 mark)

(ii) A complex number (1 mark)

(b)

(i) State three conditions to be satisfied for a function $f(z)$ to be continuous. (3 marks)

(ii) For what values of z will the function $f(z) = \frac{3z^2+1}{z^2+1}$ be continuous? (3 marks)

(c) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic. (4 marks)

(d) Evaluate $\oint_c \frac{z+4}{z^2+2z+5} dz \;\; c: |z+1| = 3$, using Cauchy's theorem. (5 marks)

(e) Find all the values of z such that $e^z = 1 + i\sqrt{3}$ (5 marks)

(f) Let $w = 3z + 4 - 5i$. Find all the values of w which corresponds to $z = -3 + i$ on the z plane. (4 marks)

(g) Solve the equation $z^4 - 16 = 0$. (4 marks)

QUESTION TWO (20 MARKS)

(a) Determine the singular points of the following function and residues at each point $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\oint_c \frac{z^2}{(z-1)^2(z+2)} dz$ where $c: |z| = 3$. (6 marks)

(b) Show that $\cos^{-1} = iln(z + i\sqrt{1-z^2})$. Hence find all solutions to the equation $\cos z = \sqrt{2}$ (7 marks)

(c) Show that $|\cosh z|^2 = cos^2 y + sinh^2 x$. (7 marks)

QUESTION THREE (20 MARKS)

(a) State the condition for integrability of $f(z)$. (2 marks)

(b) State and prove Cauchy's integral formula and give the general expression for the n^{th}derivative of $f(z)$. (7 marks)

(c) Find the image of the point $(4,3)$ on the z- plane under the transformation $w = 2z^2 + 3$. (2 marks)

(d) Integrate z^2 along the straight line OM (direct) and along an indirect path consisting of two straight line segments OL and OM, where O is the origin, M is the point $z = 3 + i$ and $L(3,0)$. Show that integral of z^2 along the two paths are equal. Hint: Sketch the region. (7 marks)

QUESTION FOUR (20 MARKS)

(a) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace equation. Hence determine the function $f(z) = u + iv$ where u is given above. (5 marks)

(b) Evaluate the points of discontinuity of

(i)$f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z = 5}{z - i}$ (4 marks)

(ii) $f(z) = \frac{z^2+1}{z^4-16}$ (4 marks)

(c) Find the fifth root of the complex number $-4 + 4i$ (7 marks)

QUESTION FIVE (20 MARKS)

(a) Find the residue of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles (7 marks)

(b) Expand the function $f(z) = (z-4)\sin\frac{1}{z+2}$ in Laurent series. (6 marks)

(c) Evaluate $\int_{(0,3)}^{(2,4)} ((2y + x^2)dx + (3x - y)dy)$ along the parabola $x = 2t$ and $y = t^2 + 3$ (7 marks)