MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER SUPPLEMENTARY/SPECIAL EXAMINATION FOR
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
BACHELOR OF SCIENCE IN CIVIL AND BIULDING ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: **TIME: 2 HOURS**

Attempt ***question one (compulsory)*** and ***any other two*** questions.

1. a) Define the following terms as used in differential equations context ;
   i. Analytic function (2 marks)
   ii. Power series solution (2 marks)
   iii. Ordinary point of a differential equation (2 marks)
   iv. Regular singular point of a differential equation (2 marks)

   b) Using the power series method solve the differential equation
   $y' - xy = 0$ composing your result to those obtained analytically. (5 marks)

   c) Determine the polynomial solution to the Legendre equation
   $(1-x^2)y'' - 2xy' + 12y = 0$ (5 marks)

   d) Using the Frobenius method solve the differential equation
   $4xy'' - 2y' + y = 0$ (5 marks)

e) Determine the Laplace transform of $f(t) = e^{at}$ where a is a constant. (3 marks)

f) Determine the half range sine Fourier series representing the function $f(x)$ defined by;

$f(x) = 1 + x \quad ; \quad 0 < x < \pi$

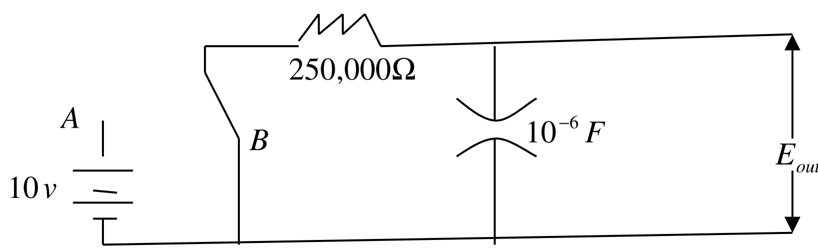
$f(x) = f(x + 2\pi)$ (4 marks)

2. a) use Laplace transforms method to solve the second order differential equation

$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t}$ given that at $t = 0$; $x = 5$ *and* $\frac{dx}{dt} = 7$ (7 marks)

b) Determine the inverse of $\frac{5s+1}{(s-4)(s+3)}$ (3 marks)

c) Suppose the capacitor in the circuit whose diagram is given below initially has zero charge and that there is no initial current. At time $t = 2\sec onds$, the switch is thrown from position B to A, held there for $1 \sec ond$, then switched back to B. Calculate the output voltage E_{out} on the capacitor. (10 marks)



3. a) use the Bessel's function identities to express $J_3(x)$ in terms of $J_0(x)$ *and* $J_1(x)$ (3 marks)

b) Evaluate $\int x^3 J_0 dx$ (3 marks)

c) Determine $J_{\pm\frac{3}{2}}$ in terms of the elementary functions $\sin x$ *and* $\cos x$ (4 marks)

d) Show that $xF(1,1,2;-x) = \ln(1+x)$ (4 marks)

e) Determine the general solution to Bessel's equation of order zero. (6 marks)

4. a) Determine the Fourier's series expansion for;

$$f(x)=\begin{cases}0 & ; -\pi \le x < \frac{-\pi}{2}\\ \cos x & ; \frac{-\pi}{2} \le x < \frac{\pi}{2}\\ 0 & ; \frac{\pi}{2} \le x < \pi\end{cases}$$ (4 marks)

b) Determine a Fourier cosine series for $f(x)=e^x$ on $(0,\pi)$ (4 marks)

c) i) Define a Dirac delta function (1 mark)

ii) State the convolution theorem of Fourier transforms. (3 marks)

iii) Given the functions $f(t)$ and $g(t)$ and their convolution is;

$$f(t)*g(t)=\int_{-\infty}^{\infty} f(x)g(t-x)dx = h(t)$$

Where $*$ denotes the convolution operation.

If $f(t)=u(t)$ and $g(t)=\begin{cases}\sec^2 t & |t|<\frac{\pi}{4}\\ 0 & otherwise\end{cases}$

Where $u(t)$ is the Heaviside function.

Show that $u(t)=\dfrac{1+\tan^2 t}{1+\tan t}$ (4 marks)

d) Find the inverse transform of $f(\omega)=\dfrac{1}{2\pi(a+j\omega)^2}$ where $a>0$ (5 marks)

5. a) Solve $\dfrac{\partial^2 u}{\partial x \partial y}=\sin x \sin y$ subject to the conditions;

$$\frac{\partial u}{\partial x}\left(x,\frac{\pi}{2}\right)=2x$$ (5 marks)

$$u\ (\pi, y)=2\sin y$$

b) Use the method of separation of variables to find the solution of the equation;

$\dfrac{\partial^2 u}{\partial x^2}=\dfrac{1}{k}\dfrac{\partial u}{\partial t}$ Which satisfies the conditions;

i. $u(0,t)=u(a,t)=0 \quad \forall\ t \ge 0$

ii. $u(x,0)=x \quad for\ 0 \le x \le a$ (15 marks)