MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SUPLEMENTARY EXAMINATION FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
BACHELOR OF SCIENCE IN BUILDING AND CIVIL ENGINEERING

ECU 300: ENGINEERING MATHEMATICS IX

DATE: **TIME: 2 HOURS**

Attempt ***question one (compulsory)*** and ***any other two*** questions.

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Given that $\vec{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$

i. Show that $\vec{F}$ is a conservative force field. (3 marks)

ii. Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$ (3 marks)

iii. Find the scalar potential (4 marks)

b) Evaluate $\oint_C \vec{F}.d\vec{r}$ around a closed curve C bounded by $y^2 = x$ and $x^2 = y$, if $\vec{F} = (x-y)i + (x+y)j.$ (5 marks)

c) Find the area of the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 1$ (5 marks)

d) A vector field $\vec{F} = yi + 2j + k$ exists over a surface S defined by $x^2 + y^2 + z^2 = 9$ bounded by $x = 0,\ y = 0,\ z = 0$ in the first octant. Evaluate $\int \vec{F}.d\vec{S}$ over the indicated surface. (10 marks)

QUESTION TWO (20 MARKS)

a) Consider $I = \int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} xy^2 dxdy$

i. State the order of integration, giving reasons (2 marks)

ii. Evaluate I (4 marks)

b) Evaluate the line integral $\oint xydx + (2x - y)dy$ round the region bounded by the curve $x^2 = y$ and $x = y^2$ by use of Green's theorem. (7 marks)

b) Verify the divergence theorem for $\vec{F} = 4xi - 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$ (7 marks)

QUESTION THREE (20 MARKS)

a) Evaluate the double integral $\iint_R \frac{x}{y} dxdy$ where R is the rectangle $|x| \le 1$, $1 \le y \le 2$

(8 marks)

b) Verify Stoke's Theorem for the vector function $F = yi + zj + xk$, for the surface S where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary (12 marks)

QUESTION FOUR (20 MARKS)

a) Find

i. A unit normal to the surface $\phi(x, y, z) = xy - z^2 = 0$ at $(1,4,-2)$ (2 marks)

ii. The equation to the normal line. (3 marks)

iii. The equation to the tangent plane at $(1,4,-2)$ (3 marks)

b) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^2$ from $t = 1$ to $t = 2$ (6 marks)

c) Represent $\vec{F} = zi - 2xj + yk$ in cylindrical coordinates. Thus determine F_ρ, F_ϕ, F_z. (6 marks)

QUESTION FIVE (20 MARKS)

a) Find

i. The velocity and acceleration of a particle which moves along a curve
$r = 2i\sin 3t + 2j\cos 3t + 8kt$ (3 marks)

ii. Find also the magnitudes of the velocity and acceleration. (3 marks)

iii. c) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction
$2i - j - 2k$ (4 marks)

b) If $\vec{A} = 2i - 3j + 4k$, $\vec{B} = i - 2j - 3k$ and $\vec{C} = 2i + j + 2k$ then show that the scalar triple product in absolute represents the volume of a parallelepiped with sides $\vec{A}, \vec{B}$ *and* $\vec{C}$ (10 marks)