



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2022/2023

**SECOND YEAR FIRST SEMESTER SUPPLEMENTARY EXAMINATIONS FOR
THE DEGREE OF BACHELOR OF SCIENCE IN**

SMA 260: PROBABILITY AND STATISTICS I

DATE : _____ TIME: 2 HOURS

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) Define a random variable. (1 mark)
- b) Consider the random variable x with the cumulative distribution function (c.d.f),
- $$F(x) = \begin{cases} 1 - e^{-x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$
- i. What is the p.d.f of x ? (2 marks)
- ii. Use the p.d.f obtained to find the $p(x > 10)$. (2 marks)
- iii. Use the c.d.f to find the $p(x > -8)$. (2 marks)
- c) Let x have the p.d.f, $f(x) = \begin{cases} 2(1-x) & ; 0 < x < 1 \\ 0 & ; elsewhere \end{cases}$. Obtain $E(6x + 3x^2)$. (3 marks)

- d) As part of an air pollution survey, an inspector decides to examine the exhaust of 12 out of 36 vehicles of a company. If 8 vehicles of the company emit excessive amount of pollutant, determine the probability that 3 of such vehicles will be included in the sample examined by the inspector. (3 marks)
- e) A fair six faced die is tossed once. Determine:
- i. Its probability distribution. (2 marks)
 - ii. The mean and variance. (2 marks)
- f) The operational lifespan of a given brand of desktop computers has been found to be normally distributed with a mean of 4.8 years and a standard deviation of 1.6 years.
- i. Determine the proportion of the desktop computers that will have a lifespan of between 3.8 years and 6.6 years. (2 marks)
 - ii. If these desktop computers have a warranty period of 2 years, determine the proportion of original sales which will require replacement through this warranty. (2 marks)
 - iii. If the manufacturer of these desktop computers wants only 5% of the computers to be replaced through this warranty, determine the warranty period that should be set to achieve this. (2 marks)
- g) Given the mgf of a random variable x as $e^{2(e^t-1)}$;
- i. Prove that the x has mean equal to the variance. (4 marks)
 - ii. Write down the p.m.f of x and prove that it satisfies the conditions necessary for a p.m.f. (3 marks)

QUESTION TWO (20 MARKS)

- a. A continuous random variable x is given by the function, $f(x) =$
- $$\begin{cases} \frac{2}{9}(9x - x^2 - 18) & ; \quad 3 \leq x \leq 6 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$
- I. Show that the function is a probability density function. (2 marks)
- II. Determine;
- i. The mean of x . (2 marks)
 - ii. The variance of x . (3 marks)
 - iii. The median of x . (3 marks)
- III. Determine the probabilities:
- i. $P(x \geq 4)$ (2 marks)
 - ii. $P(4 \leq x \leq 5)$ (2 marks)
 - iii. $P(x \leq 5)$ (2 marks)
- b. Suppose x has a pmf, $f(x) = \begin{cases} cx & ; \quad x = 1, 2, \dots, 10 \\ 0 & ; \quad \text{elsewhere} \end{cases}$.
- i. What is the value of c ? (2 marks)
 - ii. $P(x \leq 2)$ (2 marks)

QUESTION THREE (20 MARKS)

- a. A discrete random variable x has a probability function given by

X	0	1	2	3	4	5
f(x)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{2}{20}$

- i. Show that the function $f(x)$ is a probability mass function. (2 marks)
 - ii. Determine the mean and the variance of x . (3 marks)
- b. A researcher claims that only 50% of the high school students capable of passing a national examination eventually pass. Assuming that this claim is true, determine the probability that among a random sample of 8 high school students capable of passing the national examination:
- i. Exactly 4 will pass. (2 marks)
 - ii. At least 6 will pass. (4 marks)
 - iii. At most 3 will pass. (3 marks)

- iv. Between 3 and 6 inclusive will pass. (2 marks)
- c. Suppose that 10 percent of the probability for a certain distribution that is $N(\mu, \sigma^2)$ is below 60 and that 5 percent is above 90. What are the values of μ and σ ? (4 marks)

QUESTION FOUR (20 MARKS)

- a) Consider the random variable x with p.d.f $\begin{cases} \left(\frac{1}{2}\right)^{x-1} \left(\frac{1}{2}\right); & x = 1, 2, 3, \dots \\ 0 & ; otherwise \end{cases}$
- What is the $p(x > 5)$? (3 marks)
 - Find the mgf of x . (3 marks)
 - Use the mgf to compute the mean and standard deviation of x . (5 marks)
- b) Given a random variable x with p.m.f, $f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!}; & x = 0, 1, 2, \dots \\ 0; & otherwise \end{cases}$ Suppose we define a new random variable y such that $Y = 4x$. Determine the p.m.f of y . (3 marks)
- c) Suppose x is a continuous random variable having an exponential probability distribution with p.d.f given by, $f(x) = \begin{cases} \lambda e^{-\lambda x}; & 0 \leq x \leq \infty, \lambda > 0 \\ 0 & ; otherwise \end{cases}$.
- Derive the moment generating function of $f(x)$. (3 marks)
- d) Derive the first and second moment about the origin. (3 marks)

QUESTION FIVE (20 MARKS)

- a) It has been observed that a certain road has 1 pot hole for every 60 metres along the road. A random length of 360 metres is selected from a section of the road.
- Derive the Poisson probability mass function for this distribution. (2 marks)

- ii. Determine the probability that from the length selected, there are between 5 and 8 pot holes inclusive. (5 marks)
 - iii. Determine the probability that from the length selected, there are at least 4 pot holes. (4 marks)
- b) Following public outcry of teenage pregnancy in secondary schools, pregnancy test was to be carried out to students in a certain girl's school with a total population of 80 girls. A total of 12 girls were pregnant. A random sample of 20 girls was taken for testing in which the result was categorized as positive (pregnant) and negative (not pregnant). Assuming a hyper-geometric probability distribution, determine each of the following.
- i. The probability that 4 of the girls sampled tested positive. (2 marks)
 - ii. The mean and the variance of the random variable x . (3 marks)
- c) List any four characteristics of a normal distribution curve. (4 marks)