



MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR BACHELOR OF
SCIENCE IN MATHEMATICS

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 19/12/2017

TIME: 2.00-4.00 PM

INSTRUCTIONS Answer **QUESTION ONE** and any other **TWO** of the remaining questions.

QUESTION ONE: COMPULSORY (30 MARKS)

- a) If $u = (3t^2 + 1)i + (\sin t)j$ and $v = (\cos t)i + e^t k$. Find:
- $\frac{d}{dt}(u \bullet v)$ (3 marks)
 - $\frac{d}{dt}(u \times v)$ (3 marks)
- b) If $\bar{a} = 2i + 3j + 4k$ and $\bar{b} = 5i - 6j + 7k$ evaluate $\bar{a} \times \bar{b}$ (3 marks)
- c) Find the parametric equations and a vector equation of a line whose Cartesian equations are given by
- $$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5} \quad (4 \text{ marks})$$
- d) The equation of the two planes Π_1 and Π_2 are given by , $2x + 3y - 4z + 1 = 0$ and $3x - 5y + 6z + 8 = 0$ respectively. Calculate the angle between the two planes. (5 marks)
- e) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i + 6j + 7k$ (5 marks)
- f) Find the length of the arc of the curve $\bar{r} = \cos t i + e^t \sin t j + e^t k$ from $0 \leq t \leq \pi$ (7 marks)

QUESTION TWO (20 MARKS)

a) If $\vec{a} = 2i - 3j + 4k$, $\vec{b} = 5i - 6j + 7k$ and $\vec{c} = 2i + j + 2k$.

Find

i. $\vec{b} \times \vec{c}$ (3 marks)

ii. $\vec{a} \cdot (\vec{b} \times \vec{c})$ (3 marks)

iii. $\vec{a} \times (\vec{b} \times \vec{c})$ (3 marks)

iv. $\vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ (6 marks)

b) Determine the value of m such that the vectors $\vec{a} = 2i + j + 4k$, $\vec{b} = 3i + 2j + mk$ and $\vec{c} = i - 4j + 2k$ are coplanar (5 marks)

QUESTION THREE (20 MARKS)

Consider the surface $\vec{r} = ui + vj + (u^2 + v^2)k$

a) Find the vector in the direction of the tangent to the curve $u = u_0$ on the surface. (2 marks)

b) Find the vector in the direction of the tangent to the curve $v = v_0$ on the surface. (2 marks)

c) Determine a vector normal to the surface. (4 marks)

d) What is the unit vector normal to the surface. (2 marks)

e) Find the equation to the tangent plane at the point P on the surface where $u = -1, v = 1$. (5 marks)

f) Equation to the normal line at P . (5 marks)

QUESTION FOUR (20 MARKS)

a) Determine the value of p so that $\vec{a} = 2i + 3j + k$ and $\vec{b} = pi - 2j - 2k$. (2 marks)

b) Find

i. A unit normal to the surface

$\phi(x, y, z) = xy - z^2 = 0$ at the point $(1, 4, -2)$ (5 marks)

ii. The equation to the normal line (2 marks)

iii. The equation to the tangent plane at $(1, 4, -2)$ (3 marks)

c) If $\vec{a} = a_1i + a_2j + a_3k$, $\vec{b} = b_1i + b_2j + b_3k$, and $\vec{c} = c_1i + c_2j + c_3k$.

Show that $\bar{a} \times (\bar{b} \times \bar{c}) = \bar{b}(\bar{a} \cdot \bar{c}) - \bar{c}(\bar{a} \cdot \bar{b})$ (8 marks)

QUESTION FIVE (20 MARKS)

For a space curve

$\bar{r} = l \cos t \bar{i} + l \sin t \bar{j} + m t \bar{k}$ where l and m are constants determine the following

- a) The unit tangent $\bar{T}$ (3 marks)
- b) The curvature K and the radius of curvature ρ (5 marks)
- c) The unit normal $\bar{n}$ (3 marks)
- d) The unit binormal $\bar{b}$ (4 marks)
- e) The Torsion τ and the radius of the Torsion σ (5 marks)