MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE

SMA 334: FLUID MECHANICS II

DATE: 24/4/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTION: *Answer Question* ***ONE*** *which is compulsory*

QUESTION ONE (30 MARKS)

a) Given the stream functions and velocity potentials for an incompressible fluid in two-dimensional flow;

$$v_x = \frac{\partial \Psi}{\partial y}, v_y = -\frac{\partial \Psi}{\partial x} \quad \text{and } v_x = \frac{\partial \phi}{\partial x}, v_y = \frac{\partial \phi}{\partial y}$$

Show that for a potential flow; the Laplace's equations given below are satisfied

i) $\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = \mathbf{0}$ (3 marks)

ii) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ (3 marks)

b) In a two-dimensional incompressible flow, the fluid velocity components are given by $v_x = x - 4y$ and $v_y = -y - 4x$

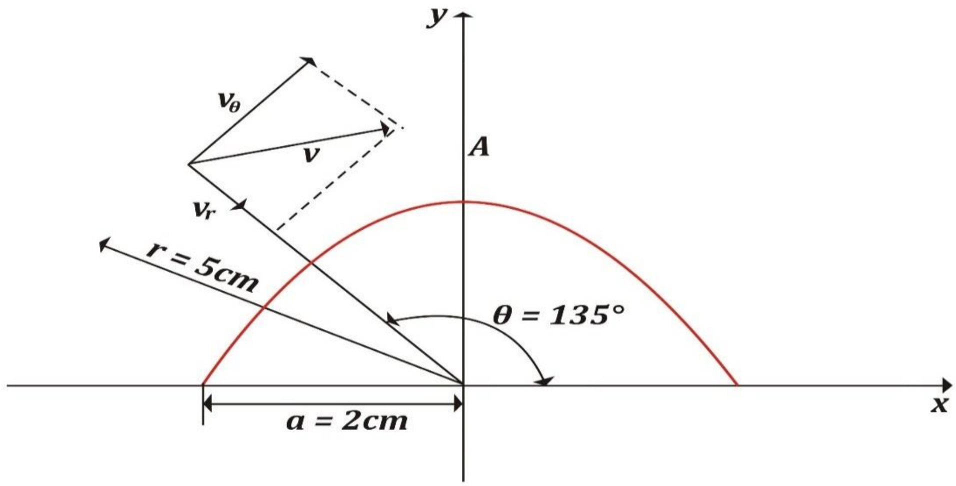
i) Show that the flow satisfies the continuity equation (4 marks)

ii) obtain the expression for the stream function (5 marks)

iii) If the flow is potential, obtain also the expression for the velocity potential (5 marks)

c) If a 4 cm diameter cylinder is immersed in a stream having a velocity of $1.0\ ms^{-1}$ as shown in figure 1 below;

Figure 1.



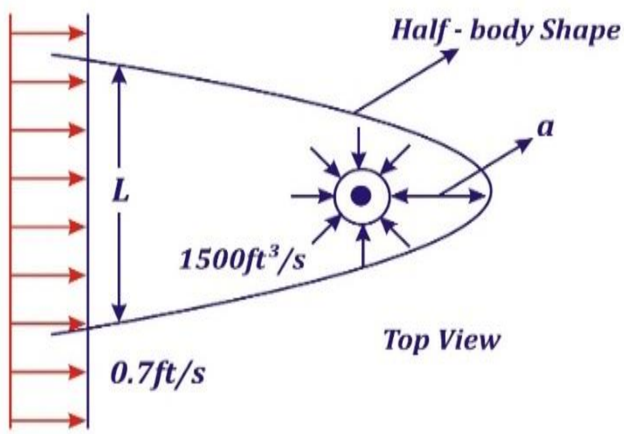
i) Determine the radial and normal components of velocity at a point on a streamline where r = 5 cm and θ = 135° measured from the positive $x - axis$. **(**5 marks)

ii) Assuming the flow is ideal, determine the pressure distribution with radial distance along the $y - axis$ (5 marks)

QUESTION TWO (20 MARKS)

a. An offshore power plant cooling-water intake sucks in 1500 ft^3/s in water 30 ft deep, as in Figure 2 below;

Figure 2.



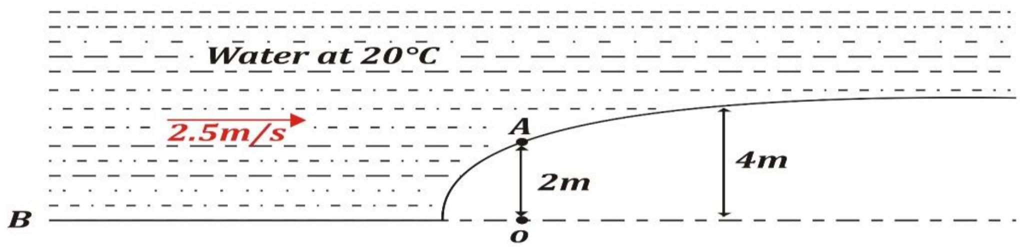
If the tidal velocity approaching the intake is $0.7\ ft/s$;

i) how far downstream does the intake effect extend (5 marks)

ii) how much width L of tidal flow is entrained into the intake? (5 marks)

b) The bottom of a river has a 4m high bump that approximates a Rankine half-body, as in Figure 3 below.

Figure 3.



The pressure at point B on the bottom is 130 kPa and the river velocity is $2.5\ ms^{-1}$. Use inviscid theory to estimate the water pressure at point A on the bump which is 2 m above point B.

(10 marks)

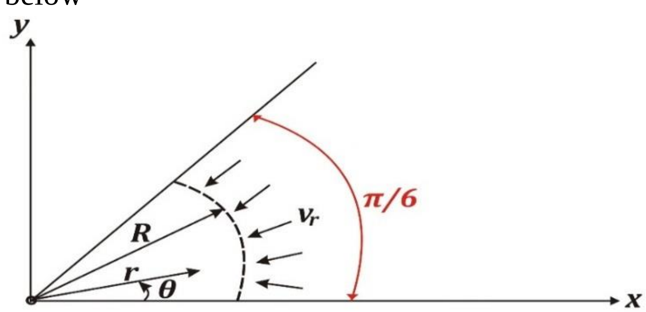
QUESTION THREE (20 MARKS)

a) The velocity distribution for the flow of an incompressible fluid is given by $v_x = 3 - x$, $v_y = 4 + 2y$, $v_z = 2 - z$. Show that this satisfies the requirements of the continuity equation in three- dimensional flow field

(4 marks)

b) A nonviscous, incompressible fluid flows between wedge-shaped walls into a small opening as shown in figure 4 below

Figure 4.



The velocity potential (in ft^2/s) , which approximately describes this flow is $\phi = -2\ln r$. determine the volume rate of flow (per unit length) into the opening. (6 marks)

c) In the ideal flow around a half-body, the free stream velocity is $0.5\ ms^{-1}$ and the strength of the source is $2.0\ m^2s^{-1}$. Determine the fluid velocity and its direction at a point, r = 1.0 m and θ = 120°

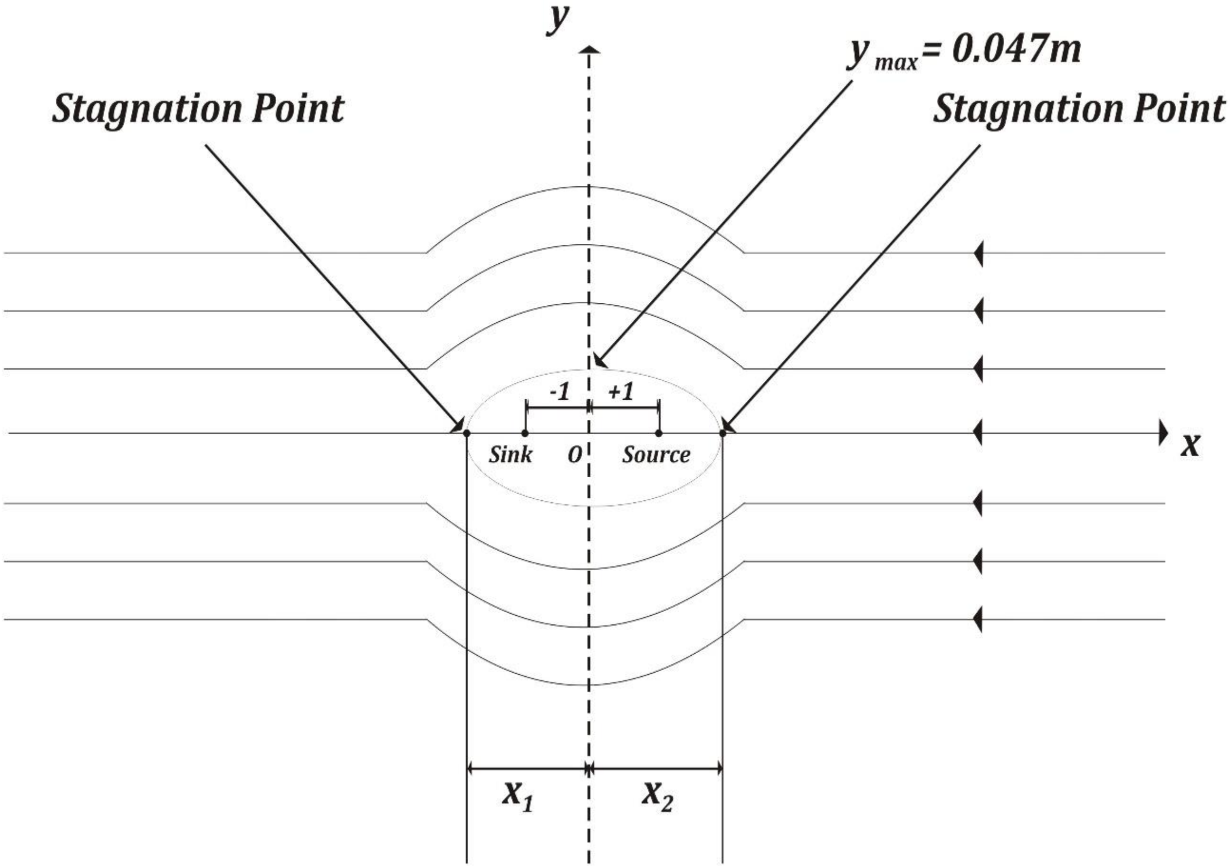
(10 marks)

QUESTION FOUR (20 MARKS)

a) Define term doublet in potential flows **(**1 mark)

b) A source of strength 10 m^2s^{-1} at $(1,0)$ and a sink of the same strength at $(-1,0)$ are combined with a uniform flow of 25 ms^{-1} in the $x-$direction as shown in figure 5 below.

Figure 5.



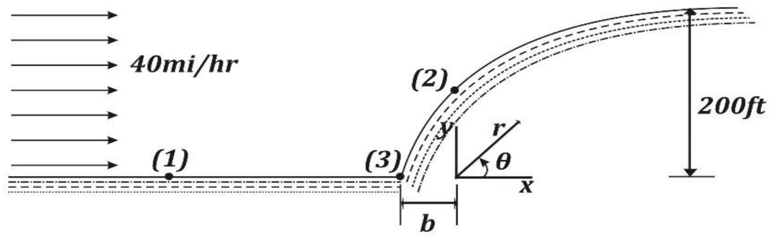
Determine;

i) the size of Rankine body formed by the flow **(**6 marks)

ii) the difference in pressure between a point far upstream in the uniform flow and the point $(1,1)$ given $y_{MAX} = 0.047m$ (6 marks)

c) A 40 mi/hr wind blows toward a hill arising from a plain that can be approximated with the top section of a half-body as illustrated in figure 6 below;

Figure 6.



The height of the hill approaches 200 ft as shown. Assume air density of 0.00238 $slugs/ft^3$;

i) What is the magnitude of the air velocity at a point on the hill directly above the origin at point (2)? (2 marks)

ii) What is the elevation of point (2) above the plain? (2 marks)

iii) What is the difference in pressure between point (1) on the plain far from the hill and point (2)? (3 marks)

QUESTION FIVE (20 MARKS)

a) Water flows through a pipeline 60 m long at a velocity of 1.8 ms^{-1} when the pressure difference between the inlet and outlet ends is 25 kNm^{-2} . What increase of pressure difference is required to accelerate the water in the pipe at the rate of 0.02 ms^{-2}? Neglect elasticity effects (8 marks)

b) A jet of water from a fixed nozzle has a diameter of 25 mm and strikes a flat plate inclined to the jet direction. The velocity of the jet is 5 ms^{-1} and the surface of the plate may be assumed frictionless.

i) Indicate in tabular form the reduction in the force normal to the plate surface as the inclination of the plate to the jet varies from 90° to 0° (6 marks)

ii) Indicate in tabular form the force normal to the plate surface as the plate velocity changes from 2 ms^{-1} to -2 ms^{-1} in the direction of the jet given that the plate is itself perpendicular to the approaching jet

(6 marks)