MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER SPECIAL/ SUPPLEMENTARY EXAMINATION
FOR BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS)
BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)
BACHELOR OF CIVIL ENGINEERING
ECU 107: ENGINEERING MATHEMATICS IV

DATE: 26/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Attempt ***question one (compulsory)*** and ***any other two*** questions.

QUESTION ONE COMPULSORY) (20 MARKS)

a) Evaluate $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x - 1}$ (3 marks)

b) Consider the function $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 2} & x \neq 2 \\ 0 & x = -2 \end{cases}$

i. Sketch the graph of the function $f(x)$ (2 marks)

ii. Determine the points of discontinuity(if any) of the function $f(x)$ (3 marks)

c) Prove that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$ (3 marks)

d) A projectile is fired straight upwards with a velocity of $400m/s$; its distance a bove the ground t sec. after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired. Calculate the maximum height achieved by the particle (3 marks)

e) Evaluate $\int \sin(3x-1)dx$ (3 marks)

f) Determine Taylors series expansion generated by the function $f(x) = \ln x$ upto the fifth term about $x = 2$ (4 marks)

g) Calculate the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ and $x = 2$ about the axis of x (4 marks)

h) Show that improper integral $\int_1^\infty \frac{1}{x} dx$ is divergent (5 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = x^3 + x^2 - 5x - 5$

i Use the second derivative test to find the local maximum and minimum of the function $f(x)$ (7 marks)

ii Discuss the concavity of $f(x)$ (5 marks)

iii. Determine the points of inflection of $f(x)$ (4 marks)

iv. Sketch the graph of the function $f(x)$ (4 marks)

QUESTION FOUR (20 MARKS)

a) A vessel containing water is in the form of an inverted hollow cone with semi-vertical angle of 30^0. There is a small hole at the vertex of the cone and the water is running out at a rate of $3cm^3$ per second $(3cm^3 / s)$. Calculate the rate at which the surface area in contact with water is changing when there is $81\pi\ cm^3$ of water remaining in the cone.

(6mks)

b) Evaluate $\int \frac{3x+7}{(x-1)(x^2+1)} dx$ (6mks)

c) Prove that $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (5mks)

d) Calculate $\int e^{\sin x} \cos x dx$ (3mks)

QUESTION FOUR (20 MARKS)

a) Determine $\frac{dy}{dx}$ for;

i.) $y = \sinh x$ (2mks)

ii.) $y = \tanh x$ (2mks)

iii.) $y = \frac{(x^2+1)^5}{\sqrt{1+x}}$ (3mks)

b) Calculate $\frac{d^2y}{dx^2}$ if;

i.) $x = t - t^3$; $y = t - t^2$ (3mks)

ii.) $x = \cos t$; $y = (1 - \sin^2 t)^{\frac{1}{2}}$ (3mks)

c) Calculate the gradient of the curve $x^3 + 2x^2y^2 - 2y + xy = 2$ at the point $(-4, 1)$ (7mks)

QUESTION FIVE (20 MARKS)

a) Evaluate

i.) $\lim_{x\to 0^-} \frac{|x|}{x}$ (3mks)

ii.) $\lim_{x\to 2^+} \frac{(x^2+2)|x|}{x^2}$ (3mks)

b) Consider the function $f(x)=\begin{cases} 7x-2 & x\geq 2 \\ 3x+5 & x<2 \end{cases}$

i) Evaluate $\lim_{x\to 2^-} f(x)$ (3mks)

ii) Sketch the graph of the function $f(x)$ (3mks)

c) Calculate the slope of the tangent line to the curve $x^3+2xy-3y^2=9$ at the point $(3,2)$

(4mks)

d) If $xy+y^2=1$ find;

i.) $\frac{dy}{dx}$ (2mks)

ii.) $\frac{d^2y}{dx^2}$ (2mks)