MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 464: DESIGN AND ANALYSIS OF EXPERIMENTS

DATE: **TIME:**

INSTRUCTIONS

1. *Answer* ***Question* 1** *and any other* ***two*** *questions.*
2. *Out of the* ***three*** *questions answered,* ***each question*** *must start on a* ***new page****.*
3. *You need the following items for this paper:*
    - *Scientific Calculator*
    - *Statistical Tables*

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Explain briefly each of the following terms as used in the design and analysis of experiments, giving examples from real life situations:

(i) Experimental unit

(ii) Treatment. (4 marks)

b) Randomisation is one of the techniques of error control in the design of field experiments. Account for the importance of randomisation. (4 marks)

c) Differentiate between an *experimental group* and a *control group* as used in the design of experiments, illustrating with an example from a real life situation. (6 marks)

d) Given a one-way classification ANOVA with the model given by $y_{ij} = \mu + t_i + e_{ij}$, and taking the various sums of squares, prove the of the following: (6 marks)

(i) Treatment sum of squares: $S_t^{\,2} = \sum_{i=1}^{k}\sum_{j=1}^{n} (\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^{k} n_i\,\bar{y}_{i.}^{\,2} - N\,\bar{y}_{..}^2$

(ii) Error sum of squares: $S_e^{\,2} = \sum_{i=1}^{k}\sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{i.}\right)^2 = \sum_{i=1}^{k}\sum_{j=1}^{n} y_{ij}^{\,2} - \sum_{i=1}^{k} n_i\,\bar{y}_{i.}^{\,2}$

e) A animal nutrition researcher carried out a study on the milk production of 4 animal feed varieties A, B, C and D in which a random sample of 32 cows were used with each feed given to a random sample of 8 cows for a period of one year. The mean weekly milk production in litres was then measured and recorded. After statistical analysis, he derived the following ANOVA table.

Source of Variation	Sum of Squares SS	Degrees of Freedom	Mean Sum of Squares - MSS	Variance Ratio F	P–value
Feed	2,862.3750				0.0002116499
Error	2,901.5000				
Total					

Given the treatment means: $\bar{y}_{A.} = 42.50$, $\bar{y}_{B.} = 48.25$, $\bar{y}_{C.} = 53.00$, $\bar{y}_{D.} = 68.00$

(i) Complete the ANOVA table above, and hence using the ***F*** statistic, evaluate whether there is a significant difference in yield between the animal feeds. (4 *marks*)

(iii) Compute the critical difference between the wheat varieties, and hence identify which pair of animal feeds differ significantly. (6 *marks*)

QUESTION TWO (20 MARKS)

a) With the aid of a suitable example, differentiate between the terms *fixed effects model* and *random effects model* as used in the design and analysis of experiments. (6 marks)

b) Given a one-way ANOVA with the model given by $y_{ij} = \mu + t_i + e_{ij}$ and assuming the fixed effects model, show that:

(i) the mean sum of squares due to error (MSS_e) is an unbiased estimator of the error variance σ_e^2 ; (6 marks)

(ii) the mean sum of squares due to treatment (MSS_t) is a biased estimator of the error variance σ_e^2 . (8 marks)

QUESTION THREE (20 MARKS)

An animal breeding researcher carried out a study on the effect of exotic breeds of dairy cows on milk production. Three breeds of dairy cows were tested: local breed, cross breed and exotic breed. A random sample of 8 dairy cows was taken from each breed. The 24 cows were then kept under the same conditions and given the same feedstuff for a period of one year. The mean monthly production of milk for each of the 24 cows was recorded in litres as shown in the table below:

Cow Breed	Observations (milk yield in litres)							
Local	64	75	65	68	80	64	78	72
Cross	62	76	84	72	78	68	80	74
Exotic	96	78	92	86	90	84	94	88

Using a complete randomised design (CRD) assuming the linear additive model $y_{ij} = \mu + t_i + e_{ij}$, and using a 5% level of significance:

a) State *three* assumptions in the model.

b) Evaluate whether there is a significant difference between the breeds (treatments) in the milk production among the dairy cows. (12 marks)

c) Compute the critical difference between the breeds of cows, and identify which pair of breeds has a statistically significant difference. (6 marks)

d) Which breed of dairy cows would you recommend, justifying your choice? (2 marks)

QUESTION FOUR (20 MARKS)

A medical researcher carried out a study on the effect of anti-malarial drugs on patients to assess the duration it takes for a patient to show signs of improvement. Four types of anti-malarial drugs were tested with 5 different categories of drug dosages in millilitres. A random sample of 20 malaria patients was taken and 5 patients randomly assigned to each drug. Each dosage was randomly assigned to 4 patients with each patient given a different drug. The duration in hours a patient took to show signs of improvement was recorded as shown in the table below:

Dosage	10 ml	15 ml	20 ml	25 ml	30 ml
Drug A	48	50	42	56	50
Drug B	52	48	30	40	46
Drug C	45	34	32	33	46
Drug D	20	26	18	23	38

a) By using a randomised block design (RBD) with drugs as treatments and dosage as blocks, and corresponding two-way ANOVA with the linear additive model $y_{ij} = \mu + t_i + b_j + e_{ij}$, and using a 5% level of significance:

(i) Evaluate whether there is a significant difference between the drugs in their effectiveness in treating malaria. (8 marks)

(ii) Evaluate whether there is a significant difference between the dosages of the drugs in the effectiveness in treating malaria. (8 marks)

b) Based on the findings in (a) above, and justifying your recommendation;

(i) Which of the four drugs would you recommend?

(ii) Which of the five dosages would you recommend? (4 marks)

QUESTION FIVE (20 MARKS)

a) Given a one way ANOVA with its model $y_{ij} = \mu + t_i + e_{ij}$ and its corresponding total sum of squares given by $S_T{}^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2$, split or decompose the total sum of squares into its various components, and hence show that $S_T{}^2 = S_t{}^2 + S_e{}^2$ (3 marks)

b) An agricultural scientist carried out a study on the effects of three factors on maize yield: varieties of maize, fertilizer type and spacing gap. Four maize varieties, four types of fertilizer, and four spacing gaps were tested. A random sample of 16 plots was taken and 4 maize varieties randomly planted in the plots in such a way that each row and each column have only one of each maize variety forming a 4 X 4 Latin square design. Four types of fertilizers were applied in each row at random and four different spacing gaps were applied in each column at random. The yield in terms of the number of bags per plot was recorded as shown in the table below:

	Column 1	Column 2	Column 3	Column 4
Row 1	$T_3 = 58$	$T_2 = 44$	$T_1 = 50$	$T_4 = 36$
Row 2	$T_2 = 22$	$T_4 = 28$	$T_3 = 48$	$T_1 = 20$
Row 3	$T_1 = 36$	$T_3 = 54$	$T_4 = 30$	$T_2 = 24$
Row 4	$T_4 = 30$	$T_1 = 30$	$T_2 = 28$	$T_3 = 60$

Using a Latin square design (LSD) and corresponding ANOVA having a linear additive model given by $y_{ijk} = \mu + r_i + c_j + t_k + e_{ijk}$, and using a 5% level of significance:

(i) Evaluate whether there is a significant difference in the maize yield:

- between the maize varieties ; (5 marks)
- between the fertilizers; (5 marks)
- between the spacing gaps. (5 marks)

(ii) What conclusions would you draw from the findings in (a)? (2 marks)