MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SCO 210: INTEGRAL CALCULUS FOR COMPUTER SCIENCE

DATE: 20TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
*Answer question **ONE (Compulsory)** and **any other TWO** questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

(a) Evaluate the following indefinite integrals

(i) $\int x^2 \sin x \, dx$ by integration by parts. (4 marks)

(ii) $\int \frac{x^3}{x^2-3x+2} dx$ by long division and partial fractions. (5 marks)

(iii) $\int (x+7)\sqrt[3]{3-2x} dx$ by substitution. (5 marks)

(iv) $\int sin^5 x cos^2 x dx$ by appropriate substitution. (4 marks)

(v) $\int \frac{dx}{4-(x-1)^2}$ by trigonometric substitution. (6 marks)

(b) Work out the following definite integral $\int_0^{\frac{\pi}{2}} cos^2 x \, dx$ (3 marks)

(c) Find the length of the arc $x = \ln(\cos y)$ from $y = 0$ to $y = \frac{\pi}{4}$. (3 marks)

QUESTION TWO (20 MARKS)

(a) Evaluate the improper integral $\int_0^{\infty} xe^{-2x} dx$ (4 marks)

(b) Evaluate $\int \frac{x^2+2x-1}{x(2x^2+3x-2)} dx$ using partial fractions. (10 marks)

(c) Given two intersecting curves $y = x^2$ and $y = x^3$;

(i) Sketch the region R enclosed by these intersecting curves. (3 marks)

(ii) Determine the area of the enclosed region R. (3 marks)

QUESTION THREE (20 MARKS)

(a) The area enclosed by the curve $y = 4x - x^2$ and the line $y = 3$ is rotated about the line $y = 3$. Find the volume of the solid generated. (5 marks)

(b) Given the curve $y = x(x-1)(x-2)$ and the $x - axis$;

(i) Sketch the regions enclosed by the two curves between $x = 0$ and $x = 4$. (2 marks)

(ii) Find the total area of the enclosed regions. (3 marks)

(c) Obtain the reduction formulae for $I_n = \int sin^n x dx$. Hence evaluate $\int sin^4 x dx$. (10 marks)

QUESTION FOUR (20 MARKS)

(a) The price $p(x)$, in dollars, of a particular commodity is estimated to be changing at a rate $\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$ where x is the consumer demand (the number of units purchased at that price). Suppose 4 units are demanded when the price is 30 dollars per unit;

(i) Determine the price function $\mathrm{p(x)}$. (6 marks)

(ii) At what price will 3 units be demanded? At what price will no units be demanded? (2 marks)

(iii) How many units are demanded at a price of 20 dollars per unit? (2 marks)

(b) Evaluate the following integrals

(i) $\int 2x\sqrt{1+x^2}dx.$ (4 marks)

(ii) $\int \frac{1}{\sqrt{9-4x^2}}dx.$ (6 marks)

QUESTION FIVE (20 MARKS)

(a) Use the Trapezoidal and Simpson's rules with five ordinates to find approximate values for $\int_0^1 \frac{4}{1+x^2}$. Compare your results with the exact value of integral. (10 marks)

(b) Peter is considering a 5-year investment, and estimates that t years from now it will be generating a continuous income stream given by the function $50t + 3000$ dollars per year. The prevailing annual interest rate r remains fixed at 4% compounded continuously during the entire 5-year term. If the future value of the investment is given by the integral $\int_0^T f(t)e^{r(T-t)}dt$, then what will the investment be worth in time $T = 5$ years. (10 marks)