



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 330: NUMERICAL ANALYSIS

DATE: 27/7/2017

TIME: 2:00 – 4:00 PM

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (30 MARKS)

- a) Tabulate the function $2x^3 - 3x + 1$ for $x = 0(1)6$ and construct a difference table up to the 2^{nd} difference. (2 marks)

Take $x_0 = 2$ use the table to evaluate

- i. Δf_1 (1 mark)
- ii. $\nabla^2 f_2$ (1 mark)
- iii. $\delta^2 f_2$ (1 mark)
- b) Compute $\Delta(e^{2x} \log 3x)$ (5 marks)
- c) Prove that the forward difference of the quotient of two functions is given by

$$\Delta \left(\frac{f(x)}{g(x)} \right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)} \quad (5 \text{ marks})$$

- d) Ordinate $f(x)$ of a normal curves in terms of standard deviation x are given by

x	1.00	1.02	1.04	1.06	1.08
y	0.2420	0.2371	0.2323	0.2275	0.2227

Find the ordinate for standard deviation $x = 1.025$ by using Newton-Gregory forward interpolation formula. (7 marks)

- e) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(8 marks)

QUESTION TWO (20 MARKS)

- a) Show that: $\nabla = 1 - E^{-1}$ (3 marks)
- b) Construct the forward difference table, where $f(x) = \frac{1}{x}$, $x = 1(0.2)2$ correct to 4 decimal places (4 marks)
- c) Obtain the approximation of $\text{Log}(1+x)$ in the form of second degree polynomial hence evaluate $\text{Log}(1.2)$ and its maximum truncation error. (6 marks)
- d) The following table gives the temperature θ (in degrees centigrade) of a cooling body at different instant of time t (in seconds)

x	1	3	5	7	9
y	85.3	74.5	67.0	60.5	54.3

Find the approximately the rate of cooling at $t = 8$ sec (7 marks)

QUESTION THREE (20 MARKS)

- a) Derive the Newton-Cotes quadrature formula (6 marks)
- b) Find the real positive root of the equation $x^3 - 7x + 5 = 0$ by bisection method, correct to 5 places of decimal. (6 marks)
- c) How accurately should the length and time of vibration of a pendulum be measured in order that the computed value of g be correct to 0.01% (8 marks)

QUESTION FOUR (20 MARKS)

- a) Find the root of the equation $\tan x = x$ near $x = 4.5$ correct to four decimal places (5 marks)
- b) Find the polynomial of lowest degree which assumes the values 10, 4, 40, 424 and 620 at $x = -2, 1, 3, 7$ and 8 also find the value of the polynomial at $x = 6$ (5 marks)
- c) Construct the difference table for the following data tabulated function:

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	1.5431	1.8107	2.1509	2.5775	3.1075	3.7622

Use suitable Newton's interpolating polynomial to estimate $f(1.1)$ and $f(1.9)$ (10 marks)

QUESTION FIVE (20 MARKS)

- a) Evaluate the integral $\int_0^6 \frac{dx}{1+x^2}$ by:
- i. Trapezoidal rule (3 marks)
- ii. Simpson $\frac{3}{8}$ (3 marks)
- b) Deduce the error in the quotient and its absolute error. (4 marks)
- c) A function $y = f(x)$ is given in the following table

x	1	1.2	1.4	1.6	1.8	2.0
y	0.00	0.128	0.544	1.296	2.432	4.00

Find the approximate values of $f'(1.1)$ and $f''(1.1)$ (10 marks)