MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 230: VECTOR ANALYSIS

DATE: **TIME:**

INSTRUCTION: *Answer Question* ***ONE*** *which is compulsory and any other* ***TWO*** *Questions*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) If two vectors $\vec{A} = 2\hat{i} + \sqrt{3}\hat{j} - 3\hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ are inclined to each other at an angle θ. Find θ. (4 marks)

b) Find the area of the triangle with vertices at $A(1,3,2), B(2,-1,1)$ and $C(-1,2,3)$ (5 marks)

c) Find the projection of the vector $7\hat{i} + 5\hat{j} + 2\hat{k}$ on the line passing through the points $P(2,3,-1)$ and $Q(-3,-5,4)$. (4 marks)

d) Given the position vector of any point on a curve is $\vec{r} = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$. Show that $\frac{d\vec{r}}{ds}.\left(\frac{d^2\vec{r}}{ds^2} \times \frac{d^3\vec{r}}{ds^3}\right) = \frac{\tau}{\rho^2}$ where τ is torsion and ρ is radius of curvature. (5 marks)

e) Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2z$ at point $(3,-2,1)$ in the direction $3\hat{i} - 2\hat{j} + 5\hat{k}$ (4 marks)

f) If $\varphi = xy + y^2z$, and C is a contour defined parametrically by $x = t^2, y = 2t, z = t + 5$ between the points $A(0,0,0)$ and $B(1,1,1)$;

   (i) Transform $\int \varphi d\vec{r}$ along curve C into an integral involving only t. (4 marks)

   (ii) Hence, evaluate the resulting line integral obtained in f) (i) above. (5 marks)

QUESTION TWO (20 MARKS)

a) Prove that $\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}.\vec{C})-\vec{C}(\vec{A}.\vec{B})$ (7 marks)

b) Given $\vec{A} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}, \vec{B} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$, and $\vec{C} = -\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$ illustrate that $(\vec{A}\times\vec{B})\times\vec{C}$ is not equal to $\vec{A}\times(\vec{B}\times\vec{C})$. (5 marks)

c) Find a set of vectors reciprocal to the set $2\hat{\imath} + 3\hat{\jmath} - \hat{k},\ \hat{\imath} - \hat{\jmath} - 2\hat{k},\ -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ (8 marks)

QUESTION THREE (20 MARKS)

a) If $F = -2e^{-2t}\hat{\imath} + \frac{6t}{3t^2+1}\hat{\jmath} - sec^2\, t\hat{k}$, then when $t = 0$, find

(i) $\frac{dF}{dt}$ (2 marks)

(ii) $\frac{d^2F}{dt^2}$ (2 marks)

(iii) $\left|\frac{dF}{dt}\right|$ (1 mark)

(iv) $\left|\frac{d^2F}{dt^2}\right|$ (1 mark)

b) Prove the Frenet-Seret relation;

(i) $\frac{d\hat{B}}{ds} = -\tau\hat{N}$ (5 marks)

(ii) $\frac{d\hat{N}}{ds} = \tau\hat{B} - k\hat{T}$ (2 marks)

c) Determine the vector $\hat{N}$ for the space curve $\overrightarrow{r(t)} = 3(sin2t\hat{\imath} + cos2t\hat{\jmath} + \hat{k})$ (7 marks)

QUESTION FOUR (20 MARKS)

a) Determine an equation for the plane perpendicular to the vector $\vec{A} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and passing through the terminal point $\vec{B} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k}$. (4 marks)

b) Prove that;

(i) $\emptyset\vec{A} = \emptyset(\vec{\nabla}\cdot\vec{A}) + (\vec{A}\cdot\vec{\nabla})\emptyset$ (3 marks)

(ii) $\vec{\nabla}\bullet\left(\frac{\vec{r}}{r^3}\right) = 0$ (3 marks)

c) Determine the unit vector $\hat{B}$ for the space curve $\overrightarrow{r(t)} = t\hat{\imath} + t^2\hat{\jmath} + \frac{2}{3}\hat{k}$. (10 marks)

QUESTION FIVE (20 MARKS)

a) Given $A = (x + cy + az)i + (3x - y - bz)j + (4x + 2y + 9z)k$ is an irrotational vector field, find a, b, and c. (4 marks)

b) Evaluate $\iint \emptyset\, \hat{n}ds$ where $\emptyset = \frac{3}{8}$xyz and s is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 1$. (7 marks)

c) Verify Green's theorem in the plane for $\oint(xy + y^2)\, dx + x^2dy$where C is the closed region bounded by $y = x$ and$y = 2 - x^2$. (9 marks)