MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 107. ENGINEERING MATHEMATICS IV

DATE: 18/6/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Using the formal definition of a derivative, prove that $\lim_{x\to 2} 5x - 7 = 3$ (3 marks)

b) Find the equation of the straight line parallel to the line $y = \frac{1}{3}x - 3$ and passing through the point $(-1,3)$ (3 marks)

c) Find the derivatives of the following functions

i. $y = (2x+7)^3$ (2 marks)

ii. $f(x) = x^2 \cos 2x$ (3 marks)

iii. $y = \frac{\sqrt{x+5}}{x^3}$ (3 marks)

d) Find the derivative of the function $f(x) = \frac{1}{x+2}$ from the first principles (5 marks)

e) Given the function $y = \frac{x^2-x-2}{x-2}$, find

i. Its domain (1 mark)

ii. Range (1 mark)

iii. The asymptotes (3 marks)

f) Given the complex function $f(z) = z^3 + 3z - 7$, find $f'(z)$ (2 marks)

g) Find the Taylor's series expansion about $x = 0$ for the function $f(x) = e^{2x}$ (4 marks)

QUESTION TWO (20 MARKS)

a) Find the maximum and minimum of the function $f(x) = x^3 + x^2 - 2x$ and sketch it. (6 marks)

b) The position of a particle at any given time t is described by the equation $S(t) = t^3 - 4t^2 + 5t$. Find an expression for the

  i. Velocity (3 marks)

  ii. Acceleration after 3 seconds (3 marks)

c) Find $\frac{dy}{dx}$ when $x = 0$ for the equation $(x+2)3^x$ (5 marks)

d) Define what is meant by a continuous function (3 marks)

QUESTION THREE (20 MARKS)

a) Evaluate the following limits

  i. $\lim_{x\to4} \frac{x^2-5x+4}{x-4}$ (2 marks)

  ii. $\lim_{x\to\infty} \frac{5x^3-2x^2+4}{2x^3-3x+10}$ (2 marks)

  iii. $\lim_{h\to0} \frac{e^h-1}{h}$ (2 marks)

  iv. $\lim_{x\to0} \frac{\tan 5x}{4x}$ (3 marks)

b) Evaluate the following antiderivatives

  i. $\int (5x - \frac{1}{x^2})\ dx$ (3 marks)

  ii. $\int x(x^4+2)^2 dx$ (4 marks)

c) Given that $x = \cos 2t$ and $y = cos^2(2t)$, find $\frac{dy}{dx}$. (4 marks)

QUESTION FOUR (20 MARKS)

a) A curve has a parametric equations $x = t^2$ and $y = 4t$. Find the equation of the normal at $(t^2, 4t)$ (5 marks)

b) Find the equation of the tangent line to the curve $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{2}\right)$(5 marks).

c) Use Newton-Raphson method to find the real root near $x = 3$ of the equation $x^3 - 3x - 5 = 0$ correct to 3 decimal places. (5 marks)

d) Evaluate $\int_{z=0}^{z=4+2i} \bar{z}\ dz$ from $z = 0$ to $z = 4 + 2i$ along the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Using logarithmic differentiation find the derivative of

i. $y = \frac{x+2}{x^2+2x-3}$ (5 marks)

ii. $e^{xy} = x^2$ (5 marks)

b) Find the area under the curve bounded by $y = x^2 + 2x + 3$, the lines $x = 1$, $x = 4$ and $y = 6$ (6 marks)

c) Find the slope of the curve $x^2y + xy^2 = 6$ at the point $(1,2)$ (4 marks)