MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION(SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 29/4/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (COMPUSARY) (30 MARKS)

a) If $u=\left(3t^2+1\right)i+(\sin t)\,j$ and $v=(\cos t)\,i+e^t k$. Find:

i) $\frac{d}{dt}(u\bullet v)$ (3 marks)

ii) $\frac{d}{dt}(u\times v)$ (3 marks)

b) Find the parametric equations and a vector equation of a line whose Cartesian equations are given by

$$\frac{x-1}{5}=\frac{y+1}{2}=\frac{z}{-5}$$ (4 marks)

c) Find

i) A unit normal to the surface
$\phi(x,y,z)=xy-z^2=0$ at the point $(1,4,-2)$ (5 marks)

ii) The equation to the normal line (2 marks)

iii) The equation to the tangent plane at $(1,4,-2)$ (3 marks)

d) The equation of the two planes Π_1 and Π_2 are given by , $2x+3y-4z+1=0$ and $3x-5y+6z+8=0$ respectively. Calculate the angle between the two planes. (5 marks)

e) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i+6j+7k$ (5 marks)

QUESTION TWO (20 MARKS)

a) Using the formula $\overline{a}.\overline{b}=|a||b|\cos\theta = a_1b_1+a_2b_2+a_3b_3$ to calculate the angle between the two vectors $\overline{a}=2i+2j-k$ and $\overline{b}=6i-3j+2k$ (5 marks)

b) Find the length of the arc of the curve $\overline{r}=e^t\cos ti+e^t\sin tj+e^tk$ from $0\le t\le\pi$

(7 marks)

c) If $\overline{a}=a_1i+a_2j+a_3k$, $b=b_1i+b_2j+b_3k$,and $\overline{c}=c_1i+c_2j+c_3k$. Show that $\overline{a}\times(\overline{b}\times\overline{c})=\overline{b}(\overline{a}\cdot\overline{c})-\overline{c}(\overline{a}\cdot\overline{b})$ (8 marks)

QUESTION THREE (20 MARKS)

a) Determine the value of p so that $\overline{a}=2i+3j+k$ and $\overline{b}=pi-2j-2k$ are perpendicular. (2 marks)

b) Consider the surface $\overline{r}=ui+vj+(u^2+v^2)k$

i) Find the vector in the direction of the tangent to the curve $u=u_o$ on the surface. (2 marks)

ii) Find the vector in the direction of the tangent to the curve $v=v_o$ on the surface. (2 marks)

iii) Determine a vector normal to the surface. (4 marks)

c) i) Find the first fundamental form on the surface, $\overline{r}=(u+v)i+(u-v)j+avk$ (4 marks)

ii) Calculate the fundamental coefficients or the fundamental magnitudes E,F and G at the point $u=1,v=1.$ (4 marks)

iii) Write down the First Fundamental form of the surface. (2 marks)

QUESTION FOUR (20 MARKS)

a) Determine the value of m such that the vectors $\overline{a} = 2i + j + 4k$, $\overline{b} = 3i + 2j + mk$ and $\overline{c} = i - 4j + 2k$ are coplanar (5 marks)

b) Prove the Frenet-Serret formulae

i) $\frac{d\overline{T}}{ds} = K\overline{n}$ (5 marks)

ii) $\frac{d\overline{b}}{ds} = -\tau\overline{n}$ (5 marks)

iii) $\frac{d\overline{n}}{ds} = \tau\overline{b} - K\overline{T}$ (5 marks)

QUESTION FIVE (20 MARKS)

For a space curve

$\overline{r} = l\cos ti + l\sin tj + mtk$ where l and m are constants determine the following

i) The unit tangent $\overline{T}$ (3 marks)
ii) The curvature K and the radius of curvature ρ (5 marks)
iii) The unit normal $\overline{n}$ (3 marks)
iv) The unit binormal $\overline{b}$ (4 marks)
v) The Torsion τ and the radius of the Torsion σ (5 marks)