



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 332: METHODS OF APPLIED MATHEMATICS I

DATE: 16/4/2019

TIME:

INSTRUCTION:

Answer Question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) i) State the general second order linear partial differential equation (P.D.E.) (2 marks)
- ii) Hence state the three classes of that P.D.E. while giving an example equation of the canonical form in each class. (5 marks)
- b) State the Fourier Cosine series expansion of a function $f(x)$ over $[-l, l]$, together with the expressions of the Fourier coefficients. (3 marks)
- c) Reduce the heat equation $u_t = \alpha u_{xx}$ to a set of ordinary differential equations. (5 marks)
- d) i) Determine the Laplace transform of $e^{at} f(t)$ for some constant a . (2 marks)
- ii) Use the Laplace transforms to solve the equation $3y' - 4y = \sin 2t$, $y(0) = 2$. (5 marks)
- e) Given that $u = u(x, y)$, apply the change of variables $\xi = \xi(x, y)$, $\eta = \eta(x, y)$ to express the following in terms of the variables ξ, η .
- i) second order partial derivative $u(x, y)$ with respect to y only (4 marks)
- ii) second order partial derivative u_{yy} (4 marks)

QUESTION TWO (20 MARKS)

- a) Sketch the function $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$, with $f(x + 2\pi) = f(x)$ and hence determine the Fourier series expansion of this function. (9 marks)
- b) Classify the partial differential equation $x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} = e^x$, and by setting $\eta = x$ transform the P.D.E. to its canonical form. (11 marks)

QUESTION THREE (20 MARKS)

- a) Determine the inverse Laplace transform of $\frac{2}{(s+1)(s^2+2s+5)}$ using partial fractions (8 marks)
- b) Show that $L\{H(t-a)f(t-a)\} = e^{as} \overline{f(s)}$ where $H(t-a)$ is the Heaviside function (5 marks)
- c) Let the Laplace transform of a function $u = u(x, t)$ be $L\{u(x, t)\} = \int_0^\infty e^{-st} u(x, t) dt$. Determine the Laplace transform of $\frac{\partial^2 u}{\partial t^2}$. (7 marks)

QUESTION FOUR (20 MARKS)

- a) Solve $u_t = \alpha u_{xx}$, $0 < x < \pi$ subject to: $u(0, t) = u(\pi, t) = 0$, $t > 0$ to obtain the particular solution when $f(x) = x(\pi - x)$. (14 marks)
- b) Show that $\{\cos 2x, \sin 2x\}$ is an orthogonal set of functions for $x \in [-\pi, \pi]$. (6 marks)

QUESTION FIVE (20 MARKS)

- a) The displacement of a semi-infinite elastic beam is described
 $u_{tt} = a^2 u_{xx}$, $x > 0, t > 0$
by $u(0, t) = f(t)$, $\lim_{x \rightarrow \infty} \frac{\partial u}{\partial x} = 0$, $t > 0$. Solve for $u(x, t)$ by the Laplace transform method.
 $u(x, 0) = 0$, $\frac{\partial u}{\partial t} \Big|_{t=0} = 0$, $t > 0$ (10 marks)
- b) Find the characteristics of the equation $4u_{xx} + 12u_{xy} + 9u_{yy} - 9u = 9$ and reduce it to the appropriate canonical form. (10 marks)