MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (COMPUTER AND MATHEMATICS)

BACHELOR OF ARTS (MATHEMATICS AND ECONOMICS)

BACHELOR OF SCIENCE (COMPUTER AND STATISTICS)

BACHELOR OF EDUCATION (SPECIAL EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA: 335: ORDINARY DIFFERENTIAL EQUATION I

DATE: 26/7/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Attempt question ***one (compulsory***) and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Consider the differential equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$.
Show that $y=\ln x$ is its solution on $(0,\infty)$ but not a solution on $(-\infty,\infty)$ (4 marks)

b) Determine the differential equation whose solution is $y=A\cos 8x+B\sin 8x$
(3 marks)

c) An RL circuit has an electromotive force of $5volts$, a resistance of 50 $Ohms$, an inductance of one $Henry$, and no initial current. Determine the current in the circuit at any time t (4 marks)

d) Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=0 \quad y(0)=-3,\ y'(0)=-1$ (4 marks)

e) Solve the differential equation $\frac{dy}{dx}=y$, using the power series method in ascending powers of x (5 marks)

f) Define the wronskian of a set of functions (2 marks)

g) Solve the Bernoulli's equation $\frac{dy}{dx}-\frac{3}{x}y=x^4y^{\frac{1}{3}}$ (5 marks)

h) Define the Legendre linear equation of the nth order. (3 marks)

QUESTION TWO (20 MARKS)

a) Solve the differential equation $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1$
(8 marks)

b) Solve the differential equation $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=\sin 3x$ (8 marks)

c) Calculate the wronskian of the functions x^2 and $x^2\ln x$ (4 marks)

QUESTION THREE (20 MARKS)

a) Consider the equation $(3x+2y-5)dx+(2x+3y-5)dy=0$

i.) Show that the equation is homogeneous in x *and* y *only* (3 marks)

ii.) Solve the differential equation (7 marks)

b) Solve the equation $(D-1)(D+3)(D-5)y=\cosh x$ (6 marks)

c) Solve $\frac{dQ}{dt}+\frac{2}{10+2t}Q=4$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Determine the orthogonal trajectories of the family of curves $y=cx^2$. sketch the orthogonal trajectories. (7 marks)

b) Solve the differential equation $x\frac{d^2y}{dx^2}-\frac{dy}{dx}+4x^3y=x^4$ (7 marks)

c) Solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-3)y=e^{x^2}$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Show that the differential equation $\frac{dy}{dx}=\frac{3yx^2}{x^3+2y^4}$ is not exact. Hence or otherwise solve. (6 marks)

b) A metal bar at temperature of 100^0F is placed in a room at a constant temperature of 0^0F . If after $20\min$ the temperature of the bar is 50^0F . Calculate;

i.) The time it will take the bar to reach a temperature of 25^0F . (4 marks)

ii.) The temperature of the bar after $10\min$ (2 marks)

c) A certain radioactive material is known to decay at a rate proportional to a mount present. If initially there is $50 milligrams$ of the material and after two hours it is observed that the material has lost 10% of its original mass. Calculate;

i) An expression for the mass of the material remaining at any time (t) (4 marks)

ii) The mass of the material after four hours (2 marks)

iii) The time at which the material has decayed to one half of its initial mass. (2 marks)