MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE (MATHEMATICS)

SMA 332: METHODS OF APPLIED MATHEMATICS 1

DATE: 20TH JANUARY, 2025 **TIME: 2.00 PM - 4.00 PM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) State the equation representing each of the given canonical forms of 2nd order PDEs: hyperbolic, parabolic and elliptic forms. (3 marks)

b) Decompose the Laplace equation $u_{xx} + u_{yy} = 0$, $0 < x, y < L$ into ordinary differential equations (ODEs) in variables X and Y. (5 marks)

c) Sate the Fourier Cosine series expansion of a function $f(x)$ defined over the interval, $0 < x < L$. (2 marks)

d) Show that the Fourier coefficient a_n of the Fourier Cosine series, you have defined in c) above is given by; $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx, \quad n = 1,2,3,\ldots$ (5 marks)

e) Given that $u = u(x, y)$ and the change of variables $\theta = \theta(x, y), \eta = \eta(x, y)$ such that $u_x = u_\theta\theta_x + u_\eta\eta_x$, show that;
$u_{xx} = u_{\theta\theta}\theta_x^2 + 2u_{\theta\eta}\theta_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\theta\theta_{xx} + u_\eta\eta_{xx}$ (5 marks)

f) Use the First Shifting theorem to determine the inverse Laplace transform of the quotient (5 marks)

$$\frac{s+2}{s^2+2s+2}$$

g) Show that the Laplace transform $\frac{d^2y}{dt^2}$ is given by $s^2\bar{y}(s) - sy(0) - y'(0)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Given the 2nd order partial differential equation (PDE) $2u_{xx} + 2yu_{yy} + u_y = 0$

i) Determine the regions in the xy-plane where this equation is hyperbolic, parabolic and elliptic. (3 marks)

ii) For $y < 0$ transform the given equation into a hyperbolic canonical form. (10 marks)

b) Obtain the Fourier series expansion for the function

$$f(x) = 2x, \quad -\pi < x < \pi$$ (7 marks)

QUESTION THREE (20 MARKS)

a) Use partial fractions to determine the inverse Laplace transform of the quotient:

$$\frac{5s+3}{(s-1)(s^2+2s+5)}$$ (8 marks)

b) Solve the following differential equation using Laplace transform method;

$y'' + 2y' + y = 3t\,e^{-t}$ Subject to $y(0) = 4,\ y'(0) = 2$ (12 marks)

QUESTION FOUR (20 MARKS)

a) Solve the following heat distribution equation by the method of separation of variables

$$\frac{\partial u}{\partial t} = c^2\,\frac{\partial^2 u}{\partial x^2}, 0 \le x \le 2, t > 0$$

$$u(0,t) = u(2,t) = 0, t > 0 \; and \; u(x,0) = x + 2$$ (16 marks)

b) Show that the wave equation $u_{tt} = \alpha^2\, u_{xx}$ is variable separable, hence find its general solution. (4 marks)

QUESTION FIVE (20 MARKS)

a) Find the Fourier series expansion of $f(x) = x - x^2$ on the interval $-\pi \le x \le \pi$, hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi}{12}$ (15 marks)

b) Find the Laplace transform of $f(x) = \sin \alpha\, x$ (5 marks)