MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SPECIAL NEED EDUCATION)

BACHELOR OF EDUCATION (SCIENCE)

SPH 400 CLASSICAL MECHANICS

DATE: 12/8/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

Answer question ONE **(COMPULSORY)** and any other TWO questions.

Question one carries 30 marks, all other questions carry 20 marks each

QUESTION ONE (30 MARKS)

a) State the two advantages of Lagrangian mechanics over that of Newtonian mechanics. (2 marks)

b) Differentiate between the following terms

i. Curvilinear and rectilinear motion (2 marks)

ii. Translational and rotational motion (2 marks)

iii. Center of mass and center of gravity (2 marks)

c) Convert the following polar co-ordinates to Cartesian (rectangular) coordinates (4 marks)

$$\left(2, \frac{\pi}{4}\right)$$

d) Define the following terms and give their mathematical expressions

i. Torque (2 marks)

ii. Angular momentum (2 marks)

iii. Kinetic energy (2 marks)

e) Show that for a rigid body, virtual work is zero (4 marks)

f) A ball of mass M and radius R starts from rest at a height of h and rolls down a 30° slope (as shown in figure 1). Determine the linear speed of the ball when it leaves the incline. (Assume that the ball rolls without slipping). Take moment of inertia of a sphere as $I = \frac{2}{5}MR^2$. (6 marks)

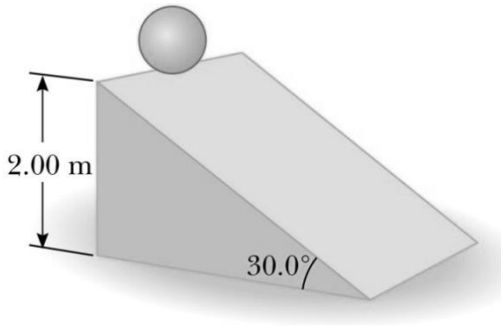


Figure 1

g) Write the lagrange's equations with both conservative and non-conservative forces (2 marks)

QUESTION TWO (20 MARKS)

a) Show that the center of mass position (in Cartesian coordinates) for the uniform solid cone shown in the figure 2 below **is** $z = \frac{3}{4}h$ (10 marks)

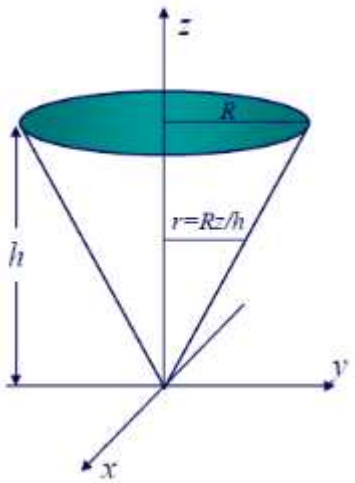


Figure 2

b) A projectile is launched with a speed of 40 m/s at an angle of 60° from the ground. Find the projectile's

i. Total time in the air (2 marks)

ii. Max height reached (2 marks)

iii. Maximum horizontal distance covered (2 marks)

c) Show that the moment of inertia of the solid cylinder whose axis of rotation is the z-axis given by $I = \frac{1}{2}MR^2$ (4 marks)

QUESTION THREE (20 MARKS)

a) State the:

i. Principle of conservation of angular momentum (2 marks)

ii. D'Alembert's principle of virtual work (2 marks)

iii. Hamilton's principle (2 marks)

b) Show that for a system of many particles, the time derivative total angular momentum equal to the externally applied torque and is given by: (7 marks)

$$\dot{L} = \sum_{\alpha=1}^{N} \vec{r}_\alpha \times \vec{F}_\alpha^{Ext} = \tau^{Ext}$$

(Assuming that the Vector $r_\alpha - r_\beta$ is aligned with the inter-particle force $F_{\alpha\beta}$)

c) A motor attached to a grindstone exerts a constant torque of 10 N-m. The moment of inertia of the grindstone is I = 2 kg-m^2. The system starts from rest. Calculate;

i. the kinetic energy after 8s (3 marks)

ii. the work done by the motor during this time (4 marks)

QUESTION FOUR (20 MARKS)

a) Figure 3 shows a ladder sliding down a wall. Find the equation of motion. (Assume the motion is in plane and the wall and the ladder are smooth. Where C(x,y) is the coordinates of the center of gravity of the ladder (10 marks)

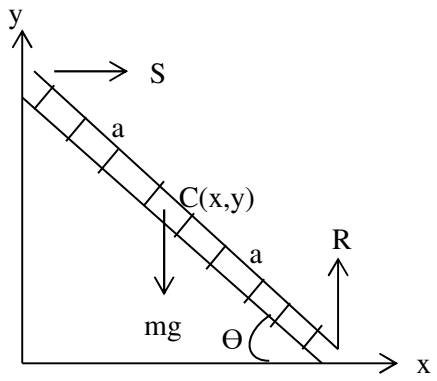


Figure 3

b) Show that the Lagrange's equation from the D'Alembert's principle is given by: (10 marks)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0$$

Where $L(q_j,\dot{q}_j,t)=T(q_j,\dot{q}_j,t)-V(q_j,t)$

QUESTION FIVE (20 MARKS)

a) The figure below show a cord wrapped round a disk and hanging box. Given that mass of the box is m =1.2kg, mass of the disk is M = 2.5kg and r = 0.2m and the cord does not slip or stretch, determine the:

(take moment of inertia of a disk $I=\frac{1}{2}MR^2$)

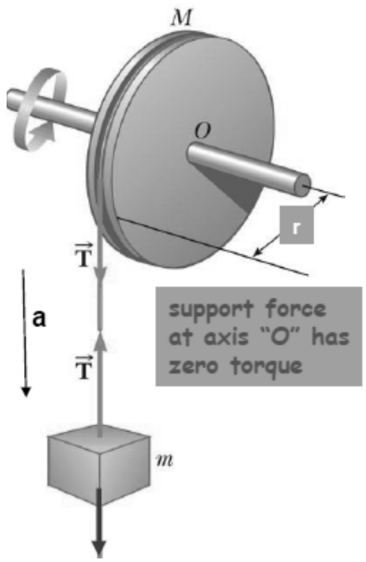
i. Angular acceleration of the disc α (3 marks)

ii. Linear acceleration a (3 marks)

iii. Tension, $\vec{T}$ (2 marks)

iv. The linear displacement of the box (2 marks)

Figure 4



b) Figure 5 shows a ball rolling without slipping down incline A, starting from rest. At the same time, a box (of equal mass as the ball) starts from rest and slides down incline B, which is identical to incline A except that it is frictionless. With proof, determine which one arrives at the bottom first? (10 marks)

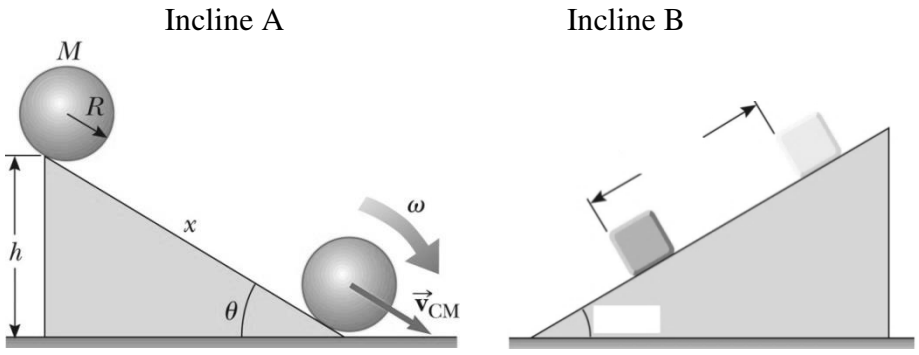


Figure 5