MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 336 ORDINARY DIFFERENTIAL EQUATIONS II

DATE: 20/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question One and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Describe the following terms as used in differential equations
   i) Analytic functions (2 marks)
   ii) Regular singular points (2 marks)

b) Evaluate the Eigen values of matrix A given that $A = \begin{bmatrix} 5 & 1 \\ 3 & 7 \end{bmatrix}$ (4 marks)

c) Reduce the initial value problem $2\ddot{x} - 5\dot{x} + x = 0$, $x(3) = 6$, $\dot{x}(3) = -1$ in to a first order matrix system. (4 marks)

d) Identify the singular points and if any, classify them given the second order differential equations
   i) $x^2y'' - xy' + y = 0$ (3 marks)
   ii) $2(x-1)^2xy'' + 3xy' + (x-2)y = 0$ (3 marks)

e) Without solving, examine the validity for the solutions to the following initial value problem $(x^2 - 4)y' + 2y = ln|8 - 4x|$; $y(4) = -2$ (5 marks)

f) Obtain the equivalent matrix system for
$$\frac{dx}{dt} - x - y = 0$$
$$\frac{dy}{dt} - 9x - y = 0$$
(3 marks)

g) Find the equilibrium points of the system of the differential equation $x_1' = 1 - x_2$ $x_2' = x_1^3 + x_2$ (4 marks)

QUESTION TWO (20 MARKS)

a) Apply the Frobenius method to find the solution of the differential equation $4xy'' + 2y' + y = 0$ about $x = 0$ (10 marks)

b) Obtain the series solution to the Bessel equation $x^2y'' + xy' + (x^2 - 1)y = 0$ (10 marks)

QUESTION THREE (20 MARKS)

a) Solve the initial value problem

$$x_1' = -5x_1 + x_2$$
$$x_2' = 2x_1 - 4x_2$$
$$x_1(0) = 2, \quad x_2(0) = -1$$ (8 marks)

b) Solve $z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - xz)dz = 0$ (6 marks)

c) By method of Frobenius, find the indicial equation and its roots for the differential equation $x^2y'' - xy' + y = 0$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Determine the ordinary and singular points for each of the following differential equations

i) $y'' - \frac{1}{x^2-9}y' + \frac{1}{x+1}y = 0$ (2 marks)

ii) $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$, p is an integer (2 marks)

iii) $xy'' + (\sin x)y' + (e^x - 1)y = 0$ (2 marks)

b) Given that $A = \begin{pmatrix} 0 & 1 \\ 8 & -2 \end{pmatrix}$, find the eigen values of At and hence find the exponential matrix e^{At} (5 marks)

c) Solve the total differential equation using the general method $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz$ (5 marks)

d) State the four types of equilibrium points as used in systems of differential equations (4 marks)

QUESTION FIVE (20 MARKS)

a) Solve the system of equations $\dot{x} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 4 & -8 & 2 \end{bmatrix}\bar{x}$ (10 marks)

b) Classify the equilibrium points of the systems below (10 marks)

i) $\dot{x} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\bar{x}$

ii) $\dot{x} = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\bar{x}$

iii) $\dot{x} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}\bar{x}$

iv) $\dot{x} = \begin{bmatrix} -4 & 3 \\ 5 & -2 \end{bmatrix}\bar{x}$