



MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

SMA 262: DIFFERENTIAL EQUATIONS

DATE: 20/4/2023

TIME: 8:30 – 10:30 AM

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE COMPULSORY (30 MARKS)

- a) Form the differential equation associated with $y^2 = 4ax$ (3 marks)
- b) Find the differential equation of the family $y = Ae^{2x} + Be^{-2x}$ (5 marks)
- c) Find the general solution of $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$ (5 marks)
- d) Apply the method of separation of variables to solve $\frac{dy}{dx} + xy = xy^3$ (7 marks)
- e) A chain coiled up near the edge of a smooth table begins to fall over the edge.

When a length x of the chain has fallen, the equation of motion is given by

$$\frac{d}{dt}(mxv) = mxg, \text{ when } m \text{ is the mass of the chain per unit length, } v \text{ is the speed,}$$

g is the acceleration due to gravity and t is the time. Show that the speed is given by

$$v = \sqrt{\left(\frac{2}{3}\right)gx} \quad (10 \text{ marks})$$

QUESTION TWO (20 MARKS)

- a) Show that

$Ax^2 + By^2 = 1$ is the solution of

$$x \left[y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right] - y \frac{dy}{dx} = 0 \quad (3 \text{ marks})$$

- b) Use the method of undetermined coefficient to solve

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 4y = 111e^{2x} \cos 3x \quad (7 \text{ marks})$$

- c) Find the orthogonal trajectories of the family

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1 \quad (10 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Solve $D^2 + 2D + 1 = x \cos x$ (6 marks)

- b) Obtain the equation of the curve satisfying this equation and passing through the

origin $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$ (6 marks)

- c) A cyclist, moving on a level road at $4m/s$ stops pedaling and free wheels to rest.

The retardation of the cycle has two components a constant $0.08m/s^2$ due to friction in the working parts and resistance of $0.02v^2/s^2$ where v is the speed in meters per second. What distance traversed by the cyclist before it comes to rest?

(8 marks)

QUESTION FOUR (20 MARKS)

- a) Find the general solution to $(D^4 - 81)y = 0$ (4 marks)

- b) Find the general solution of

$$y = x + 2 \tan^{-1} p \quad (6 \text{ marks})$$

- c) Reduce the given differential equation

$$x^2 \left(\frac{dy}{dx} \right)^2 + y(2x + y) \frac{dy}{dx} + y^2 = 0 \text{ to Clairut's form by using the substitution}$$

$y = u$, $xy = v$, and find its singular solution. (10 marks)

QUESTION FIVE (20 MARKS)

- a) Solve

$$(2x + 3y - 5) \frac{dy}{dx} + (3x + 2y - 5) = 0 \quad (10 \text{ marks})$$

- b) If the population of a country doubles in 50 years. In how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants? (10 marks)