# MACHAKOS UNIVERSITY

**University Examinations 2019/2020 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR**

**BACHELOR OF SCIENCE IN MATHEMATICS**

**BACHELOR OF EDUCATION**

**SMA 230: VECTOR ANALYSIS**

**DATE:** **TIME:**

**INSTRUCTIONS**

Attempt question ***one (compulsory***) and any other ***two questions***.

**QUESTION ONE (30 MARKS) (COMPULSORY)**

a) Consider the region bounded by the equations $y = \sin x$ and $y = 0$ from the line $x = 0$ to $x = \pi$ . Calculate the centroid (center of mass) of the region (3 marks)

b) Calculate the area bounded by the curve $y = \frac{1}{\sqrt{x}}$ , $x - axis$ and the line $x = 1$ (3 marks)

c) Evaluate $\int \left( \frac{2x^3 + 5x^2 + 8x + 9}{x} \right) dx$ (4 marks)

d) Find the length of the arc from $\theta = 0$ to $\theta = \frac{\pi}{4}$ of the curve given by $x = 3\cos\theta,\ y = 3\sin\theta.$

e) Evaluate the definite integral $\int_0^2 \cos^2 x dx$ (5 marks)

f) Sketch the curve $y = 2x^2$ from $x = 0$ to $x = 5$. Hence evaluate the area enclosed by the curve, $x -$ axis and the ordinates $x = 1$ to $x = 4$ (5 marks)

g) Find the area between the curves $x^2 = y$ and the straight line $y = 2$ (5 marks)

h) Show that the improper integral $\int_1^{\infty} \frac{1}{x^2} dx$ is convergent (5 marks)

**QUESTION TWO (20 MARKS)**

a) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ (3 marks)

b) Evaluate by adjustment of the derivative $\int \frac{x^2+x+1}{2x^3+3x^2+6x+5} dx$ (4 marks)

c) Use appropriate technique to evaluate the integral

$\int \sin^4 x dx$ (6 marks)

d) Use partial fractions to evaluate the integral

$\int \frac{3x^2-2x-7}{x^2-x-2} dx$ (7 marks)

**QUESTION THREE (20 MARKS)**

a) Prove that $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$ where $a$ and $b$ are constants (4 marks)

b) Use the fact that $\sin^2 x + \cos^2 x = 1$ to evaluate the integral

$\int \cos^3 x \sin^2 x dx$ (4 marks)

c) A particle carrying a positive electrical charge $+q$ is released at a distance $d$ from an immovable positive charge $+Q$. Calculate amount of work done on the particle as its distance increases to $2d$. (4 marks)

d) Find the surface area generated by the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$ about the axis.

(8 marks)

**QUESTION FOUR (20 MARKS)**

a) Use integration by parts to evaluate

i. $\int x^3 \ln x dx$ (5 marks)

ii. $\int x^2 e^{2x} dx$ (5 marks)

b) Draw the curve $6y^2 = x(x-2)^2$ and calculate the length of the loop of the curve $y^2 = \frac{1}{6}x(x-2)^2$ (5 marks)

c) Find the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ and $x = 2$ about the axis of $x$. (5 marks)

**QUESTION FIVE (20 MARKS)**

a) Evaluate;

i.) $\int_0^{\pi} x \sin^6 x \, cox^4 x \, dx$ (3 marks)

ii.) $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta) d\theta$ (6 marks)

b) Compute $\int_0^1 \frac{\tan^{-1} x}{x^{\frac{3}{2}}} dx$ using Simpson's rule with 10 *equal* intervals. (11 marks)