MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 4/8/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE: COMPUSARY (30 MARKS)

a) If $u = (3t^2 + 1)i + (\sin t)j$ and $v = (\cos t)i + e^t k$. Find:

i. $\frac{d}{dt}(u \bullet v)$ (3 marks)

ii. $\frac{d}{dt}(u \times v)$ (3 marks)

b) If $\bar{a} = 2i + 3j + 4k$ and $\bar{b} = 5i - 6j + 7k$ evaluate $\bar{a} x \bar{b}$ (3 marks)

c) Find the parametric equations and a vector equation of a line whose Cartesian equations are given by

$$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5}$$ (4 marks)

d) The equation of the two planes Π_1 and Π_2 are given by , $2x + 3y - 4z + 1 = 0$ and $3x - 5y + 6z + 8 = 0$ respectively. Calculate the angle between the two planes. (5 marks)

e) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i+6j+7k$ (5 marks)

f) Find the length of the arc of the curve

$\bar{r}=\cos ti+e^{t}\sin tj+e^{t}k$ from $0\le t\le\pi$ (7 marks)

QUESTION TWO (20 MARKS)

a) If $\bar{a}=2i-3j+4k$, $\bar{b}=5i-6j+7k$ and $\bar{c}=2i+j+2k$.

Find

i. $\bar{b}\times\bar{c}$ (3 marks)

ii. $\bar{a}\bullet(\bar{b}\times\bar{c})$ (3 marks)

iii. $\bar{a}\times(\bar{b}\times\bar{c})$ (3 marks)

iv. $\bar{b}(\bar{a}\bullet\bar{c})-\bar{c}(\bar{a}\bullet\bar{b})$ (6 marks)

b) Determine the value of m such that the vectors $\bar{a}=2i+j+4k$, $\bar{b}=3i+2j+mk$ and $\bar{c}=i-4j+2k$ are coplanar (5 marks)

QUESTION THREE (20 MARKS)

Consider the surface $\bar{r}=ui+vj+(u^{2}+v^{2})k$

a) Find the vector in the direction of the tangent to the curve $u=u_o$ on the surface. (2 marks)

b) Find the vector in the direction of the tangent to the curve $v=v_o$ on the surface. (2 marks)

c) Determine a vector normal to the surface. (4 marks)

d) What is the unit vector normal to the surface. (2 marks)

e) Find the equation to the tangent plane at the point P on the surface where $u=-1, v=1$.

(5 marks)

f) Equation to the normal line at P. (5 marks)

QUESTION FOUR (20 MARKS)

a) Determine the value of p so that $\overline{a} = 2i + 3j + k$ and $\overline{b} = pi - 2j - 2k$. (2 marks)

b) Find

i. A unit normal to the surface

$\phi(x, y, z) = xy - z^2 = 0$ at the point $(1,4,-2)$ (5 marks)

ii. The equation to the normal line (2 marks)

iii. The equation to the tangent plane at $(1,4,-2)$ (3 marks)

c) If $\overline{a} = a_1 i + a_2 j + a_3 k$, $\overline{b} = b_1 i + b_2 j + b_3 k$,and $\overline{c} = c_1 i + c_2 j + c_3 k$.

Show that $\overline{a} \times (\overline{b} \times \overline{c}) = \overline{b}(\overline{a} \cdot \overline{c}) - \overline{c}(\overline{a} \cdot \overline{b})$ (8 marks)

QUESTION FIVE (20 MARKS)

For a space curve

$\overline{r} = l\cos ti + l\sin tj + mtk$ where l and m are constants determine the following

a) The unit tangent $\overline{T}$ (3 marks)

b) The curvature K and the radius of curvature ρ (5 marks)

c) The unit normal $\overline{n}$ (3 marks)

d) The unit binormal $\overline{b}$ (4 marks)

e) The Torsion τ and the radius of the Torsion σ (5 marks)