



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SMA 463: TIME SERIES ANALYSIS

DATE: 8/5/2019

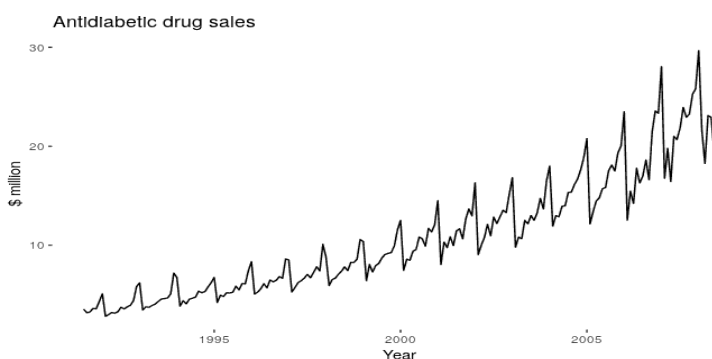
TIME: 2:00 – 4:00 PM

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Using mathematical notations, where necessary, define the following terms as used in time series analysis: -
- i) a time series (2 marks)
 - ii) IID noise (2 marks)
 - iii) ARMA(p,q) process (3 marks)
- b) What is the main purpose of time series analysis? (2 marks)
- c) State the four components of a time series. (4 marks)
- d) The graph below shows the sales of antibiotic drugs over a period of 10 years.



What statistical conclusions can you draw from the graph, based on time series analysis?

(2 marks)

- e) i) Briefly describe the process of calculating moving averages of period m . (3 marks)
- ii) Using the data given below, calculate the 5-point moving totals and moving averages.

t	1	2	3	4	5	6	7	8	9	10
X_t	12	10	11	11	9	11	10	10	11	12

(6 marks)

- f) Given an ARMA(p, q) process, show that

when $p=0$ we have an MA(q) process and

(3 marks)

when $q=0$ we have an AR(p)

(3 marks)

QUESTION TWO (20 MARKS)

- a) Define stationarity.

(2 marks)

- b) Consider the series defined by the equation

$$X_t = Y_t + \Theta Y_{t-1}, \text{ for } t = 0, \pm 1, \pm 2, \dots$$

where $\{Y_t\} \sim WN(0, \sigma^2)$ and $|\Theta| < 1$ is a real-valued parameter.

Show that $\{X_t\}$ is stationary.

(8 marks)

- c) Define the term linear process as used in time series analysis.

(3 marks)

- d) Prove that, if $\frac{1}{2q+1} \sum_{j=-q}^q Y_{t-j} \approx 0$ (usually what happens if we choose q “large enough”) this filter will allow a linear trend function $m_t = \beta_0 + \beta_1 t$ to pass without distortion. (7 marks)

QUESTION THREE (20 MARKS)

- a) State two limitations of the method of moving averages. (2 marks)
- b) The following data refers to the UK outward passenger movement by sea.

	Year 1				Year 2				Year 3			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4
No. of passengers (millions)	2.2	5.0	7.9	3.2	2.9	5.2	8.2	3.8	3.2	5.8	9.1	4.1

Using least squares regression, calculate a trend component for each time point given.

(8 marks)

- c) Let the random walk $\{S_t; t = 0, 1, 2, \dots\}$ be obtained by summing up random variables. The random walk has zero mean and is obtained by defining

$$S_0 = 0 \text{ and}$$

$$S_t = X_1 + X_2 + \dots + X_t \text{ for } t = 1, 2, \dots$$

$$\nabla S_t = S_t - S_{t-1} = X_t$$

$$\{X_t\} \sim \text{IID}(0, \sigma^2)$$

Determine whether the random walk is stationary or not.

(10 marks)

QUESTION FOUR (20 MARKS)

- a) Define the terms

MA(q) and AR(p), as used in time series analysis.

(4 marks)

- b) Show that the MA(q) process given by

$$X_t = Z_t + \Theta_1 X_{t-1} + \Theta_2 X_{t-2} + \dots + \Theta_q X_{t-q} \text{ is a linear process.}$$

(4 marks)

- c) Consider the series defined by the equation

$$X_t = Y_t + \Theta Y_{t-1}, \quad t = 0, \pm 1, \pm 2, \dots$$

where $\{Y_t\} \sim \text{WN}(0, \sigma^2)$ and $|\Theta| < 1$ is a real-valued parameter. Determine the

- i) autocovariance (ACVF) function

(5 marks)

- ii) autocorrelation (ACF) function.

(4 marks)

- d) Sketch the correlogram of the equation in (c) above. (3 marks)

QUESTION FIVE (20 MARKS)

- a) Define the term causality as used in time series analysis. (3 marks)

- b) Show that

- (i) a MA(1) process with representation

$$X_t = Z_t + \Theta_1 X_{t-1} + \Theta_2 X_{t-2} + \dots + \Theta_q X_{t-q} \text{ is causal} \quad (4 \text{ marks})$$

- (ii) an AR(1) process define by $X_t = Z_t + \phi X_{t-1}$ is causal (4 marks)

- c) Given that the difference operator ∇ is defined by

$$\nabla X_t = X_t - X_{t-1}. \text{ Determine } \nabla^2 X_t \quad (4 \text{ marks})$$

- d) Define a correlogram. (2 marks)

- e) State three conditions that are necessary for stationarity. (3 marks)